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Two Sided Matching: A Study of Underemployment

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Two-Sided Matching: A Study of Underemployment

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Abstract

This paper lays out a general equilibrium framework to study and predict the underemployment rate. A heterogeneous labor market in which both workers and firms differ in skill type is considered and calibrated to simulate both unemployment and underemployment rates. Qualitatively, the model's predictions match the empirical evidence suggesting underemployment decreases with increases to unemployment insurance. Quantitatively, the one-shot foundation limits the model's predictive capabilities.

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1 Introduction

With over 30 percent of the U.S. workforce considered underemployed, underemployment is an area ripe for study and one that lends itself perfectly to heterogeneous matching theory. Measured here as the share of the college educated workforce working jobs that do not require a college degree, skill-based mismatches are the at the heart of underemployment's definition. The goal of this paper is to explore the relationship between underemployment and unemployment insurance under a general equilibrium framework. To this end, I construct a labor market search-and-match model that distinguishes between worker skill types and job skill requirements in order to measure the effects of the replacement rate on the underemployment rate. An empirical analysis reveals that the replacement rate has a weakly positive relationship with the unemployment rate and a significantly negative relationship with the underemployment rate. Stratifying by sex and marital status further suggests that the group whose underemployment rates are most impacted by increases to the replacement rate are married men followed by single females. This is an interesting conclusion and a potential area of future research.

Directionally, the model's predictions are in line with the data in looking at both the positive increase in unemployment and decrease in underemployment in response to increases in the replacement rate. To gauge the magnitudes of the model's predictions, the model is calibrated, with parameters assigned values based on both outside literature and observed data. Predictions are compared to the historical period they simulate to measure the model's quantitative predictive capabilities. While the model's underemployment predictions are clustered around the range observed in the data, there is little evidence that the predictions are correlated with the data. With

regards to unemployment, the model vastly over-predicts unemployment and again little evidence is seen of correlation between the predictions and actuality. There are several potential explanations for the model's lack of quantitative predictive capabilities. While the exact reason is not pinned down here, it is likely in part due to the one-period framework in which the model is based. This clear deviation from real-world labor markets gives give rise to numerous potential issues.

Several recent papers are related to my work. The primary contribution I hope to make to the existing literature is the addition of a general equilibrium model aimed at predicting the under-employment rate. It is first useful to place my model in the context of the general literature on matching among heterogeneous agents. Burdett and Coles (1999) offer an instructive framework for heterogeneous two-sided matching in looking at both marriage and employment markets. They draw parallels between the problems faced by a worker looking for a job (and a firm looking for a worker) to a single person looking for a marriage partner. This problem of long-term partner matching is two-sided. While a worker is evaluating a potential employer, the employer is evaluating the worker. For both sides, outside options are considered in offering a job and in accepting a job offer. If there is a strong labor market the firm has greater leverage, and for the worker, they would be more prone to accepting offers given the high supply of workers. In much of the prior literature surrounding labor market studies, homogeneity is the standard assumption with regards to employers and workers within labor markets. Burdett and Coles show that heterogeneity can be used to add valuable insights to labor economics and specifically search models.

In this spirit, Albrecht and Vroman (2002) consider a labor market with endogenous skill re-

quirements. The model I construct is motivated from their work. Functionally, I use a similar definition of skill within my model and also consider an equilibria with two worker types and two job types. Other literature relying on this same foundation include McKenna (1996) and Gautier (2002) which look at education's relationship to unemployment and search externalities respectively. In building a labor market matching model with heterogeneous skill types across workers and employers, Albrecht and Vroman examine the various explanations that have been offered for increases in wage inequality since the Great Depression. While that is not the purpose of my paper, I am able to use a similar theoretic foundation under a one-shot horizon to study underemployment drivers. Barnichon and Zylberberg (2019) propose a markedly different way of modeling underemployment in studying why the underemployment rate grows during recessions. They develop a search model with endogenous ranking of skilled-applicants under a stochastic environment with worker heterogeneity within skill groups. Their model diverges significantly from the one presented here in that worker skill types are unobserved by firms in an attempt to define underemployment based on skill rather than education.

It is also useful to contextualize my work within the empirical literature on underemployment. Abel and Deitz (2016) document the jobs held by underemployed college graduates and show that relatively few recent grads were working in low-skill services jobs despite being underemployed. My definition of underemployment, the share of employed college graduates working jobs that do not require a college degree, is the same as found here and in much of the empirical literature. Beaudry, Green and Sand (2016) examine a question raised by Abel and Deitz of why high underemployment persisted in the 2000s. They argue a shift in the skill-demand among employers

around 2000 caused high-skill workers to move down the occupational ladder and, in so doing, displacing lower-educated workers. Meanwhile, Maynard and Feldman (2011) take a greater look at the psychological and social challenges held by underemployed workers. They find consistently negative effects of holding a job that pays too little or underutilizes one's skill. Additionally, using survey data, they suggest that those in inadequate jobs have lower job satisfaction and that wage penalties among recent college graduates as a result of underemployment can depress earnings for a significant amount of time.

One more related area of interest is that of career mobility. Where my work only discusses job matching, this helps to answer the implications of being underemployed within the labor market post-matching. It is important to note that career mobility is inherently dynamic, so while my paper has relevance to the field it is only at a point in time, specifically job consummation. Sicherman and Galor (1990) analyze theoretically and empirically the role of occupational mobility in the labor market and find that one return to higher education is a greater probability of occupational upgrading both within and across firms. They also find that returns to education diminish with time spent in the labor market. Blazquez and Alba-Ramirez (2004) look to Spain in a study concerning workers who report having skills or qualifications for a more demanding job than the one currently held. Their results suggest overeducated workers generally have shorter job tenure than appropriately educated workers. They also show evidence supporting Sicherman and Galor with regards to overeducated workers having a higher probability of promotion. One area of fragmentation within the literature is with regards to wage catch-up for overeducated workers. Where Blazques and Alba Ramirez (2004), along with much of the existing literature, find evidence that overeducated

workers experience greater catch-up over time in wages relative to better matched workers, Buchel and Mertens (2000) show evidence that is more in line with Abel and Deitz (2016) in that overeducated workers experience relatively slower wage growth. Blazques and Alba-Ramirez (2004) also document how an increase of the education level of the Spanish workforce has been accompanied by a rise in jobs for higher educated workers. The model constructed in this paper makes the same prediction in that in response to an increase in high-skill workers, firms will post more vacancies targeted towards these high-skill workers.

2 The Model

2.1 Basic Assumptions.

Working from Abrecht and Vroman's infinite horizon model, I construct a one-period model in which all workers begin unemployed. The labor market is heterogeneous in supply, such that some workers can be either low- or high-skill, and in demand, with employers posting vacancies that require either a low- or high-skill level. Jobs are such that low-skill jobs can be filled by either type of worker, but high-skill job vacancies may only be filled by high-skill workers. As given by Albrecht and Vroman, an illustrative example is a pilot can fly a 747 or deliver drinks to the passengers, whereas a flight attendant can deliver drinks but cannot fly the plane. By my definition, the pilot is a high-skill worker and flying the plane is a high-skill job. Workers of a high-skill type working a low-skill job do not produce more than a worker of lesser skill at the same position. Workers undergo an undirected search process in which the probability that a worker matches with a job position is given endogenously as a function of vacancy postings.

The distribution of skills across workers is given exogenously by $\gamma = \frac{U_L}{U_L+U_H}$ where U_i are unemployed workers in the type $i \in \{L, H\}$ market. It is simplest to normalize the measure of workers to one such that $U_L, U_H \in [0, 1]$ and $U_L + U_H = 1$. This allows for the construction of the primary two endogenous variables. $\theta = \frac{V_L+V_H}{U_L+U_H}$ is a measure of labor market tightness in which V_i is a vacancy of skill type $i \in \{L, H\}$ and $\phi = \frac{V_L}{V_L+V_H}$ is the share of vacancies that are low skill.

After matching, high-skill jobs with high-skill workers produce output Z_H whereas low-skill jobs, with workers of either skill type, produce output Z_L where $Z_H > Z_L > 0$. Similarly, the cost of a filled job for high-skill firms is W_H and the cost for low-skill firms is W_L where W_i is the wage paid to the worker. As matching is not directed, unemployed workers match with vacancies randomly according to the matching function $m(U, V)$ where U is the number of unemployed workers and V is the number of vacancies. Additionally, it is assumed that the matching function is characterized by constant returns to scale, thus giving

$$m(U, V) = m\left(1, \frac{V}{U}\right) U = m(\theta)U, \text{where } \theta = \frac{V}{U} = \frac{V_L + V_H}{U_L + U_H}$$

This allows for the characterization of the job findings rate $f = m(\theta)$ and the job filling rate $q = \frac{m(\theta)}{\theta}$.

2.2 Match Formulation and Wages

The probability that a worker of low-skill type matches with a firm is given by the share of low-skill vacancies times the job-finding rate, $\phi m(\theta)$, and if matched they earn the wage W_L. If they do

not match they earn b, which is a measure of both unemployment benefits and the overall gain in leisure time. For high-skill workers, the probability of matching with a high-skill firm, and earning wage W_H, is $(1-\phi)m(\theta)$. Thus, the worker's value functions are given by:

$$J_L = \phi m(\theta) W_L + (1 - \phi m(\theta))b \tag{1}$$

$$J_H = (1-\phi)m(\theta)W_H + \phi m(\theta)W_L + (1 - m(\theta))b \tag{2}$$

On the firm side, I assume the marginal cost of posting a vacancy of either skill type is increasing. This is done for technical reasons and ensures free-entry yields a competitive market that features underemployed workers. For low-skill firms, the probability of matching with a worker is given by $\frac{m(\theta)}{\theta}$ and, if a match is made, firms earn $Z_L - W_L$ where Z_L is the output of a worker at a low-skill firm. High skill firms can only match with high-skill workers, so their probability of matching is given by $\frac{(1-\gamma)m(\theta)}{\theta}$ and, upon matching, they earn $Z_H - W_H$. Finally, with the marginal cost of posting a vacancy in either skill-type market being κV_i, the firm's value functions are given by:

$$\pi_L = -\kappa V_L + \frac{m(\theta)}{\theta}(Z_L - W_L) \tag{3}$$

$$\pi_H = -\kappa V_H + \frac{(1-\gamma)m(\theta)}{\theta}(Z_H - W_H) \tag{4}$$

Wages are determined by Nash Bargaining in which workers and firms negotiate over how to split the surplus that is output created minus worker alternative, or $Z_i - b$. With worker bargaining power given by χ, Nash Bargaining achieves:

$$W_H = \chi Z_H + (1-\chi)b \tag{5}$$

$$W_L = \chi Z_L + (1 - \chi)b. \tag{6}$$

It is important to note that while high-skill workers in high-skill jobs earn strictly greater wages than low-skill workers in low-skill jobs, workers of either skill type earn the same wage working low-skill jobs.

2.3 Equilibrium

Equilibrium is found by solving for our endogenous variables ϕ and θ. This is achieved by assuming a free-entry condition such that the expected values of entering the job searching market for a firm of either skill type is zero, or that $\pi_L = \pi_H = 0$. If expected profit of market entry were greater than zero, we would see an increase in firm job postings which would drive up the cost of market entry until π_i is back to zero. Similarly, if expected utility of entry were less than zero, firms would exit the market until π_i is once again back to zero. Thus, we can rewrite equations (3) and (4) to find

$$\kappa V_L = \frac{m(\theta)}{\theta}(Z_L - W_L)$$

$$\kappa V_H = \frac{(1-\gamma)m(\theta}{\theta}(Z_H - W_H).$$

To obtain results in closed form, I assume a Cobb-Douglas matching function such that $m(U, V) = V^{\alpha}U^{1-\alpha}$ where $\alpha \in [0, 1]$ is the elasticity of matches with respect to vacancies. In substituting this matching function into our free entry equations, algebraic manipulation yields equilibrium levels

of V_H and V_L which in turn allows for inference of θ and ϕ.

$$\theta = \frac{(Z_H + Z_L + b(\gamma - 2) - Z_H\gamma)\left(\frac{(Z_H+Z_L+b(\gamma-2)-Z_H\gamma)^{\alpha-1}(1-\chi)}{\kappa}\right)^{\frac{1}{2-\alpha}}}{(Z_H - b)(1-\gamma)} \tag{7}$$

$$\phi = \frac{Z_L - b}{(Z_H - b)(1-\gamma) + Z_L - b} \tag{8}$$

Note that we have $\theta > 0$ and $\phi \in [0, 1]$. For both, we are able to sign these given that $Z_H > Z_L \geqslant 1$ and $b, \gamma, \chi \in [0, 1]$.

2.4 Equilibrium Implications

It is now useful to discuss a few implications of this equilibrium in looking at comparative statics. Unemployment, or $\mathbb{U}$, can be solved for within the model as $1 - \theta^\alpha$. Recall that the job-filling rate is given by $m(\theta)$ where $m(\theta) = m(\frac{V_H+V_L}{U_H+U_L})$. With U normalized to one, the Cobb-Douglas matching function give us $m(\theta) = \theta^\alpha$. Thus, as this is the job filling rate, we must have $1 - \theta^\alpha$ as the unemployment rate post-matching. I find $\frac{\partial \mathbb{U}}{\partial b}, \frac{\partial \mathbb{U}}{\partial \gamma}, \frac{\partial \mathbb{U}}{\partial \kappa}, \frac{\partial \mathbb{U}}{\partial \chi} > 0$ while $\frac{\partial \mathbb{U}}{\partial Z_H}, \frac{\partial \mathbb{U}}{\partial Z_L} < 0$. Meanwhile, underemployment within the model is given simply by ϕ. This makes sense as search is undirected and the probability that a worker of high-skill type matches with a low-skill firm, or in other words the probability that a high-skill worker is underemployed, is $\phi m(\theta)$, the job-filling rate times the share of low-skill firms. I find $\frac{\partial \phi}{\partial b}, \frac{\partial \phi}{\partial Z_H} < 0$ while $\frac{\partial \phi}{\partial Z_L}, \frac{\partial \phi}{\partial \gamma} > 0$.

As seen in the comparative statics, unemployment is increasing in the replacement rate while underemployment is decreasing. It is worth looking at the mechanisms by which this is happening within the theoretic framework. With regards to unemployment, the general thinking is that higher

unemployment insurance discourages job search, however, although qualitatively this produces the same effect, that is not what is happening here. Within the model, increasing unemployment benefits raises workers outside options thereby reducing match surplus ($Z_i - b$). This induces firms to post fewer vacancies, which lowers the job finding rate $m(\theta)$ and raises unemployment. With regards to underemployment, the model predicts that underemployment will fall with unemployment insurance. Note that high skill matches create greater surplus for firms as $Z_H - b > Z_L - b$. Thus, when b increases, the surplus proportionally has gone down by more in the low-skill market. This causes firms to find it optimal to change the mix of job postings more towards the high-skill market in response to an increase in b which lowers the underemployment rate ϕ. The assumption of b being flat across workers brings gives rise to a potential issue in relating the model to the real world, where unemployment benefits are indexed to worker wage. This would imply that mechanism driving underemployment's decrease in response to unemployment benefits within the model does not exist in the real world. However, unemployment benefits have a ceiling which will disproportionately effect high-skill workers who earn more. Thus, increases in b should still lower the surplus in the low-skill market by a relatively greater amount than in the high-skill market and the mechanism in the theoretic framework driving underemployment changes is still plausible.

This analysis contains several implications that are in line with real world assumptions. Firstly, we see that unemployment is increasing in b, a measure of unemployment benefits; γ, the share of workers who are low-skill; χ, worker bargaining power; and κ, the cost of posting vacancies. Meanwhile, unemployment is decreasing in worker productivity of both skill types. All of these predictions make intuitive sense and are in line with other literature[1]. The model has now es-

[1]Effects of the replacement rate on unemployment, while intuitively significant, are actually not as strong as one

tablished qualitative predictions regarding both unemployment and underemployment that can be tested against empirical data. Specifically, the model predicts that unemployment is increasing in the replacement rate and decreasing in workforce education, while underemployment is decreasing both in the replacement rate and workforce education. With regards to all four of these qualitative predictions, the model is in line with the observed data as seen later in Section 3.2.

3 Empirical Evidence

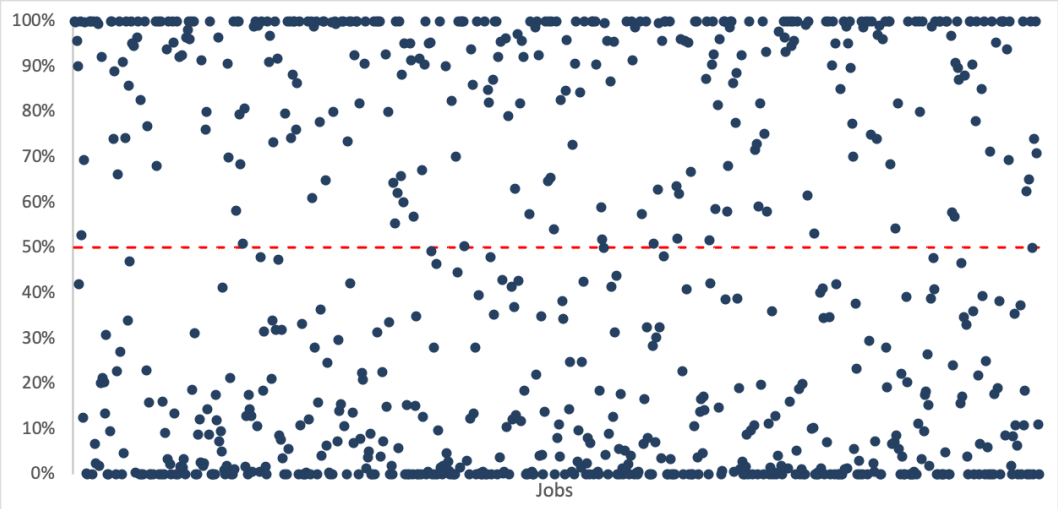
3.1 Data Description

Data is gathered from three primary sources. I obtained individual level data from the CPS regarding demographic information, employment and educational attainment. This was then aggregated on the state-level over time into a panel structure. To distinguish between college and non-college jobs, I rely on the Department of Labor's O*NET database. O*NET contains occupation-level data for hundreds of occupations collected via interviews of current workers along with input from professional job analysts on an array of job requirements. To determine whether an occupation requires a college degree, I use the following question from O*NET's Education and Training Questionnaire: "If someone were being hired to perform this job, indicate the level of education that would be *required*?" Respondents (employers) then select from twelve detailed education levels, ranging from less than a high school diploma to post-doctoral training. I consider college education to be a requirement if over 50 percent of the respondents working in that occupation indicated that at least a bachelor's degree was necessary to perform the job. This 50 percent threshold brings up a natural issue as it ignores heterogeneity within occupations. It's possible that for a

may think as seen both in Section 3.2 in this paper and elsewhere in the literature

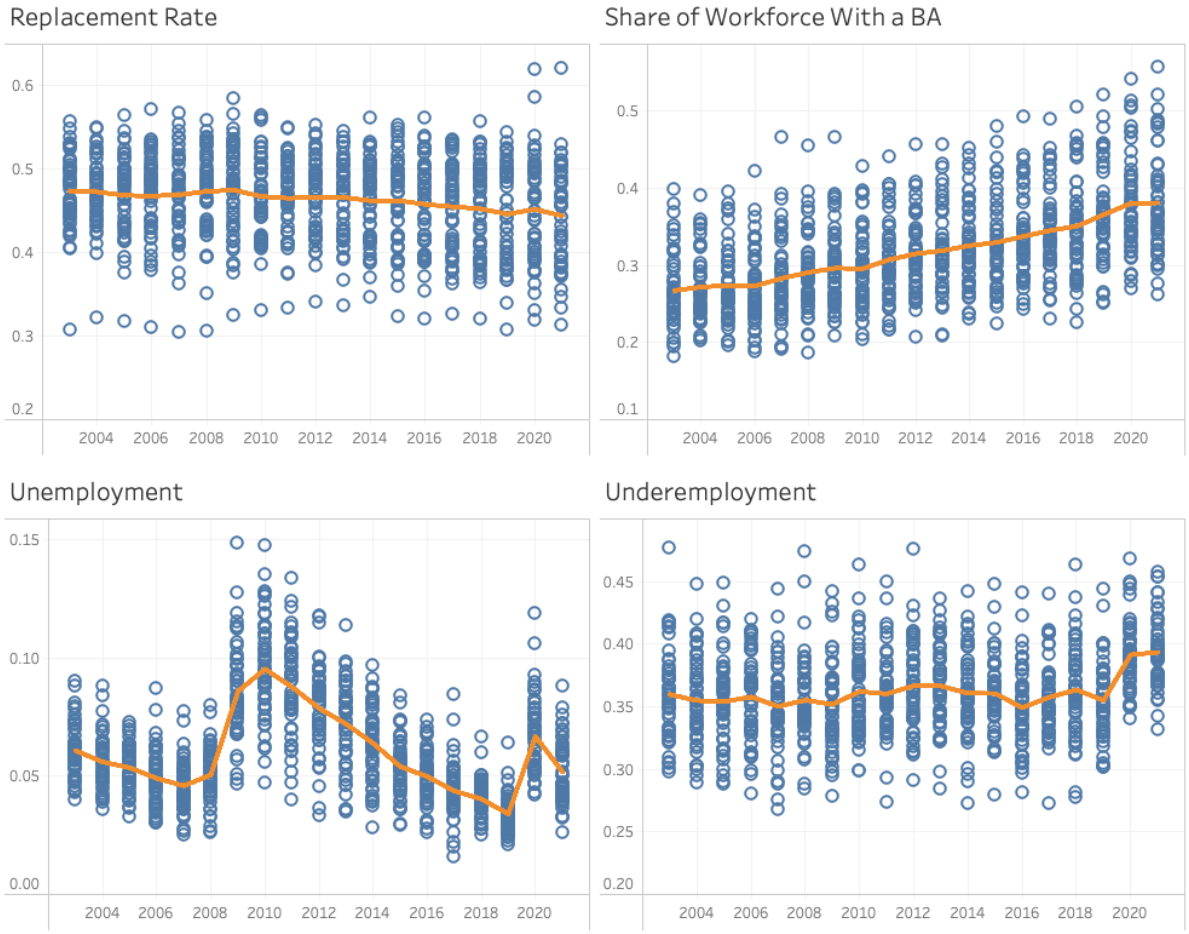
given occupation, some employers require a college degree and some do not. However, in practice, few occupations in the O*NET database are clustered around 50 percent. For most occupations, respondents either overwhelmingly felt that a bachelor's degree is required for the job or not. This distribution is given in figure 1 which pictures the percentage of respondents for a given job who would require a bachelor's degree for someone applying to that position.

Figure 1: Share of Employers Who Believe Job Requires a Bachelor's Degree



Using the individual-level CPS data in tandem with the O*NET database allows for the generation of underemployment statistics. Unemployment and Underemployment trends, variation across states, and moving averages are pictured in figure 2. Also pictured is the replacement rate over the time period and the share of workers within the state that have obtained a bachelor's degree. Replacement rate data is sourced from the U.S. Department of Labor.

Figure 2: Summary Statistics



Among all four variables, high variation is seen across states. The most noticeable trend is an increase the percentage of the workforce with a bachelor's degree. A slight decline in the replacement rate from 48 percent to 39 percent is seen over the time frame. Looking at underemployment, average underemployment over the period is 37.6 percent and there is clear variation across states. Additionally, there is little evidence that underemployment is correlated with general business cycle fluctuations as unemployment is. Finally, a natural question is if unemployment is correlated with either underemployment or workforce education level. The correlation coefficients are $-.03$ and $-.11$ respectively. This is pictured below in figures 3 and 4.

Figure 3: Scatter Plot of Underemployment and Unemployment

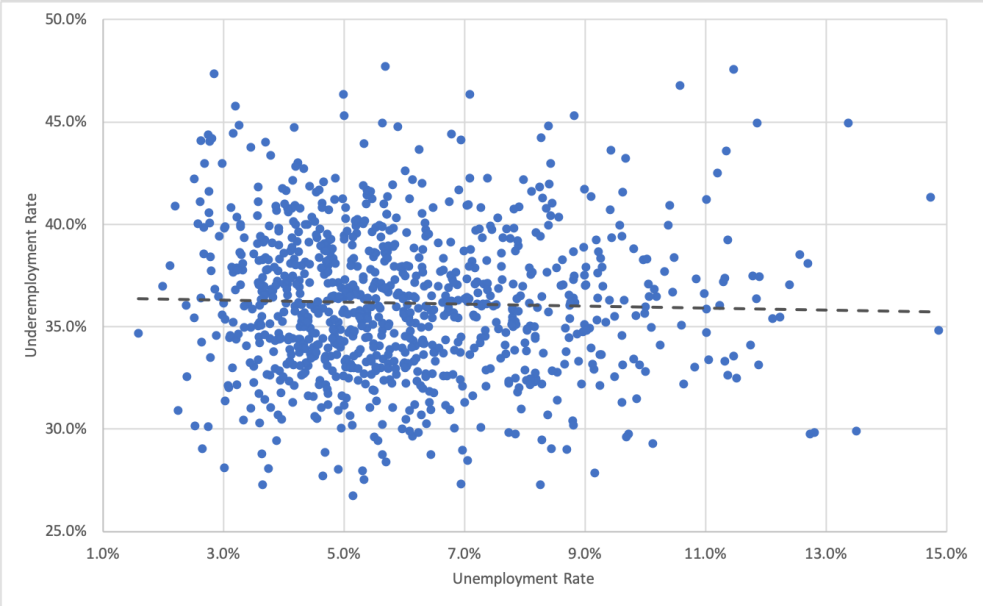


Figure 4: Scatter Plot of Underemployment and Workforce Education Level

3.2 Empirical Results

Within the data, I test the relationship between the replacement rate and both unemployment and underemployment while controlling for the education level of the workforce. The following model

is used:

$$\mathcal{Y}_{st} = \beta_0 + \beta_1 \textit{Replacement Rate}_{st} + \beta_2 \textit{Workforce BA Share}_{st} + \alpha_s + \alpha_t + \epsilon_{st} \tag{9}$$

where $\mathcal{Y} \in \{\textit{Underemployment Rate, Unemployment Rate}\}$. Pictured in figure 5 are the outcomes from this model for unemployment (top) and underemployment (bottom) respectively.

Figure 5: Regression Results

Dependent Variable: Unemployment Rate				
	(1)	(2)	(3)	(4)
Replacement Rate	-0.027**	-0.032**	.040*	.037*
	(1.349)	(1.339)	(2.157)	(2.187)
Share of Workforce with a BA		-0.049***		-0.043
		(0.011)		(0.030)
Constant	7.260***	9.026***	4.178***	5.456***
	(0.629)	(0.734)	(1.062)	(1.533)
Included Controls				
State Effects	No	No	Yes	Yes
Year Effects	No	No	Yes	Yes
Observations	949	949	949	949
R-squared	0.004	0.026	0.732	0.733
Robust standard errors in parethenthesis				
*** p<0.01, ** p<0.05, * p<0.1				

Dependent Variable: Underemployment Rate				
	(1)	(2)	(3)	(4)
Replacement Rate	.065***	.095***	-0.052*	-0.068**
	(2.181)	(2.178)	(3.069)	(2.913)
Share of Workforce with a BA		-0.056***		-0.237***
		(0.017)		(0.044)
Constant	33.147***	35.175***	38.440***	45.562***
	(1.017)	(1.193)	(1.622)	(2.053)
Included Controls				
State Effects			X	X
Year Effects			X	X
Observations	949	949	949	949
R-squared	0.009	0.020	0.204	0.227
Robust standard errors in parenthesis				
*** p<0.01, ** p<0.05, * p<0.1				

For each table, column (1) is the baseline case with no controls. Column (2) controls for the share of workforce that has obtained a bachelor's degree, and columns (3) and (4) replicate the first

two but control for state and time fixed effects.

Looking at the upper table concerning unemployment, a significant negative relationship is seen which is made more significant when controlling for education level. This makes intuitive sense as increases to the replacement rate raise workers outside options to employment and would thus increase unemployment. However, while this relationship is negative across states, a weakly positive relationship is observed once state and time fixed effects are considered (columns (3), (4))[2]. Despite the initial intuition, this is actually in line with much of the existing literature. Eugster (2015) examines the effects of a higher replacement rate on unemployment rate and duration within Switzerland. Using a difference-in-difference approach, Eugster found that a six percentage point increase in the replacement rate resulted in a 3.1 percent increase to unemployment duration but no significant increase to unemployment. Similarly, Howell and Rehm (2009) consider the validity of higher unemployment benefits as the conventional explanation of high European unemployment relative to America. They argue that in a behavioral framework, workers suffer substantial disutility from unemployment. Similar to the results presented in figure 5, they find that much of the conventional explanation is driven by differences across countries rather than within. They present the stylized fact that countries with higher unemployment generally have higher unemployment benefits but argue that this is not a causal relationship. Indeed, in looking within countries over time, there is little evidence supporting the argument that increasing unemployment insurance causes increased unemployment rates in significant manner.

[2]A natural question is if there lag such that increases to the replacement rate are not felt in the immediate future. Although not shown here, data used for this study does show any evidence of a lag within three years

Turning to the bottom table regarding underemployment, similar differences are seen in looking across states versus within. While columns (1) and (2) present evidence that increases to the replacement rate raise underemployment, this relationship turns negative once controlling for fixed effects and especially after controlling for both fixed effects and education level as seen in column (4). Qualitatively, in looking at the results concerning both unemployment and underemployment's relationship to workforce education and the replacement rate, this is the same result obtained by the theoretic model. Section 3.3 takes a quantitative approach by calibrating the model to judge its predictions versus the observed data.

Before doing so, it is worthwhile to explore if the results obtained concerning underemployment differ by sex and marital status. Figure 6 below gives the output from the prior regression model (equation 9) stratified by sex and marital status.

Figure 6: Regression Results by Group

Dependent Variable: Underemployment Rate				
	(1)	(2)	(3)	(4)
	M. Men	S. Men	M. Females	S. Females
Replacement Rate	-0.113**	-0.049	-0.031	-0.953*
	(5.562)	(6.363)	(3.960)	(4.969)
Share of Workforce with a BA	-0.173***	-0.168**	-0.113**	-0.052
	(0.057)	(0.068)	(0.055)	(0.060)
Constant	46.702***	42.703***	40.875***	41.279***
	(3.532)	(3.466)	(2.482)	(3.304)
Included Controls				
State Effects	X	X	X	X
Year Effects	X	X	X	X
Observations	949	949	949	949
R-squared	0.126	0.171	0.053	0.067
Robust standard errors in parentheses				
*** p<0.01, ** p<0.05, * p<0.1				

Note that unlike figure 5, this only presents results controlled for workforce education and state

and year fixed effects. We see that the relationship between the replacement rate and underemployment rate is negative for each group. However, this relationship is the strongest among married men. This is also the group with the largest underemployment rate within the sample at 37.59 percent. The group with the lowest underemployment rate is single females at 34.58 percent. The heightened underemployment seen among men is a result also obtained by Abel and Deitz (2016) who find that male graduates are 3 percent more likely than females to be underemployed. There is no clear reason why underemployment among married men would exhibit the largest response to changes in the replacement rate, nor why the group with the second largest response is single females. While significance levels are boarderline, this trend, if seen elsewhere, certainly merits further exploration in other work.

3.3 Calibration

This section parameterizes the model to match the time-series data by state in order to simulate and predict underemployment over the historical period. These predictions are then measured against the data. As seen in the prior section, the model's qualitative predictions established in Section 2.4 match the data with regards to how changes in the replacement rate and workforce education level impact unemployment and underemployment. A quantitative approach is now used to see if the predicted magnitudes of change match as well. Recall equations (7) and (8) concerning the unemployment rate $(1-\theta^{\alpha})$ and underemployment rate (ϕ) respectively.

$$\theta = \frac{(Z_H + Z_L + b(\gamma-2) - Z_H\gamma)\left(\frac{(Z_H+Z_L+b(\gamma-2)-Z_H\gamma)^{\alpha-1}(1-\chi)}{\kappa}\right)^{\frac{1}{2-\alpha}}}{(Z_H-b)(1-\gamma)} \tag{10}$$

$$\phi = \frac{Z_L - b}{(Z_H - b)(1-\gamma) + Z_L - b} \tag{8}$$

To calculate values for θ and ϕ, seven parameters must be calibrated: high-skill worker productivity Z_H; low-skill worker productivity Z_L; the replacement rate b; worker bargaining power χ, elasticity of matches with respect to vacancies α; the share of workforce that is low-skill γ; and the cost of vacancy posting κ. Beginning with parameter values set outside of the model, I assume that α and χ are invariant to time and state variation. This is a standard assumption as there is little evidence in the literature suggesting either worker bargaining power or the elasticity of matching with respect to vacancies fluctuates by state or has changed significantly over time. α is calibrated to be .72 in following Shimer (2005). In studying the cyclical behavior of unemployment and vacancies, Shimer measures the job-finding rate within the workforce over 1950-2005 to estimate a matching function $m(u, v)$. In a similar fashion to the model presented in this paper, Shimer imposes that the matching function be Cobb-Douglas, or that $m(u, v) = u^{\alpha} v^{1-\alpha}$. The observed job-finding rate and labor market tightness are then used used to calculate $\alpha = .72$. Whereas α is found within the literature, χ is found by solving the following system of equations and assigning mean parameter values as seen in the data:

$$\begin{gathered} \tfrac{\textit{Firm Profit}}{\textit{Output}} = \gamma\big(1 - \tfrac{W_L}{Z_L}\big) + (1-\gamma)\big(1 - \tfrac{W_H}{Z_H}\big) \\ \tfrac{W_H}{W_L} = \tfrac{\chi Z_H + (1-\chi)b}{\chi Z_L + (1-\chi)b} \end{gathered} \tag{11}$$

Firm profit over output represents profit's share of output and $\frac{W_H}{W_L}$ is the wage premium earned by high-skill workers as seen below in calibrating Z_H. Note that wages are a function of χ so while not seen, χ is embedded in the top equation. To solve this, I first isolate Z_H in the second equation and then substitute it into the first. While mean parameter values of $\gamma = .68$, $b = .46$,

and $\frac{W_H}{W_L} = 2.06$ are all available within the data set, I rely on Barkai (2016) for profit's share of output, which is found to be equal to $.10$, or 10 percent. Given the mean parameters listed along with Barkai's calculation, I obtain $\chi = .83$.

Now turning to parameters that are allowed to fluctuate across states over time, I first calibrate κ, the cost of posting vacancies, using both outside literature and the observed data. Hagedorn and Manovskii (2008), in building a search-model to generate the observed business-cycle-frequency fluctuations in unemployment and vacancies, calculate a steady-state capital flow cost of posting a vacancy to be .474, or 47.4 percent of the average weekly labor productivity. Thus, I set $\kappa = .474(\gamma Z_L + (1-\gamma)Z_H)$ as the share of workers producing Z_L are γ and the share of workers producing Z_H are $1-\gamma$. Next, the replacement rate b and share of low-skill workers γ are pulled directly from the data for each observation (observations being a given state in a given year). Without loss of generality, Z_L is normalized to one and Z_H is solved for using the difference in skill-group earnings to reflect a college wage premium. Specifically, I select Z_H for each state over time such that $\frac{W_H}{W_L}_{Data} = \frac{W_H}{W_L}_{Model}$. As wages are given by:

$$W_H = \chi Z_H + (1-\chi)b$$

$$W_L = \chi Z_L + (1-\chi)b$$

we have

$$\frac{W_H}{W_L} = \textit{Wage Ratio(WR)} = \frac{\chi Z_H + (1-\chi)b}{\chi Z_L + (1-\chi)b}.$$

With Z_L normalized to 1, we can solve for Z_H to find

$$Z_H = \frac{WR * W_L - (1-\chi)b}{\chi}.$$

After calculating the wage ratio as the mean wage earned by college-educated workers over the mean wage earned by non-college-educated workers at every point in the panel data set, we can calibrate Z_H, finalizing the calibration strategy for the non-fixed variables. All seven parameters seen in equations (7) and (8) are now accounted for, and I can quantify underemployment and unemployment within the model. After predicting an unemployment and underemployment rate for each observation, these values are compared against the actual data points in figures (7) and (8) on the following page. With regards to both unemployment and underemployment, there is no evidence that the model's prediction is correlated with the data. The model predicts unemployment rates that are on average 867 percent greater than in reality and underemployment rates that are on average 19 percent lower. Given the proposed calibration strategy, in order for predicted unemployment rates to be in the 3 to 10 percent range, as most of the observed data are, replacement rates would have to be around $-.8$, which is obviously impossible. While predictions for unemployment are far higher than in reality, underemployment rates are predicted to be within the same range as in reality despite the lack of correlation with the observed data.

3.4 Discussion

It is important to note, the International Labour Organization defines two broad types of underemployment: time-related underemployment and inadequate employment situations. Underemployment is considered to be time-related if, during the reference period, the worker was willing to

Figure 7: Unemployment Calibration

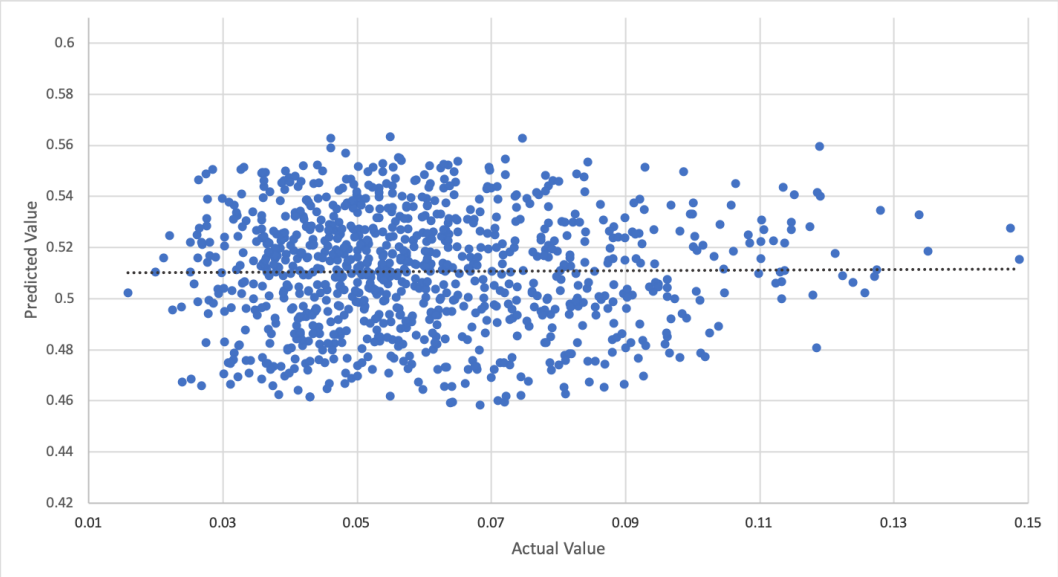
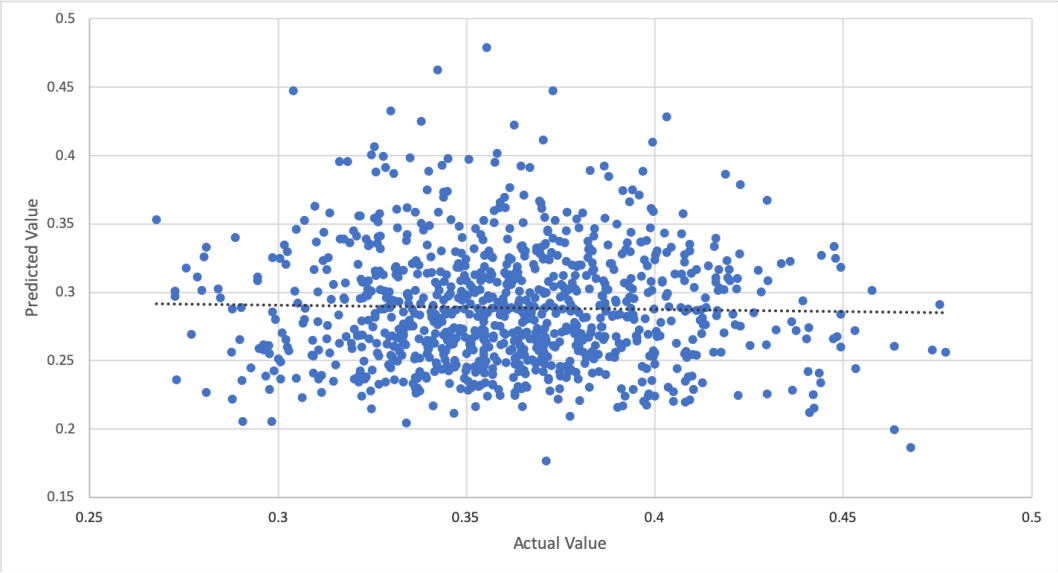


Figure 8: Underemployment Calibration



work additional hours, was available to work additional hours, and had worked fewer hours than a predetermined threshold relating to working time. Meanwhile, underemployment is classified as inadequate employment situation if it is skills-related or income-related. This paper only considers skill-related underemployment within inadequate employment situations. However, there are issues that arise when using educational requirement of an occupation as determined by professional job analyst and current workers (as seen in the O*Net database) compared with individual educational attainment as a proxy for skill mismatches. Firstly, this assumes that all jobs within the same occupation have the same educational requirements. However, this is addressed above in figure 1 which shows that heterogeneity in job requirements is not prevalent enough within occupations to raise major issues. The bigger concern is conflating educational mismatching and skill mismatching. Although this does present concerns over how underemployment is calculated both within this paper and throughout the literature, there seems to be limited solutions to systematically calculating underemployment as skill-mismatches within the workforce. Due to this, underemployment, outside of its formal definition, is generally considered to be education-based mismatches rather than skill-based.

There are a few potential reasons for the lack of correlation between the model's predictions and observed data. One assumption that deviates in an obvious way from real world markets is the one-period horizon. In the model presented in this paper, all workers begin unemployed and all firms begin with zero labor supply. Firms then judge market conditions (workers skill-type distribution, wages, worker bargaining power, vacancy posting cost, etc.) and offer a mix of high- and low-skill vacancies. Workers then go about an undirected search process and fill some of these

vacancies. This one-shot setting limits predictive capabilities in a few ways. Namely, there are no flow probabilities of workers gaining employment in the next period, which would be part of their expected utility function as is the case in the real world. Similarly, unemployment benefits do not run out, which would also impact worker decision making. For these reasons and others, using a one one-period model is an obvious abstraction from the real world which could explain the lack of correlation. This paper is grounded in a one-period model in the interest of both simplicity and an effort to better match how data was collected. As individuals were not tracked over time, the belief was that a one-period setting would better match the data which is inherently static.

Another potential explanation is that the model constructed here assumes total vacancy costs are equal to $\frac{\kappa}{2}V^2$, which yields a marginal cost of vacancy posting κV. Another approach that may be more in line with reality is assuming total costs equal κV which would yield a marginal cost of κ. The clear difference between the two is that the marginal cost of posting vacancies in this model is increasing rather than constant. Thus, the costs for a firm to post any vacancy will be greater than the prior posting. Other similar search-models are able to assume constant marginal costs to vacancy posting without suffering from indeterminancy issues, as is the case here, due to their infinite-horizon setting. This model also assumes that, for the sake of simplicity, b, or replacement rate, does not differ by skill-type. The replacement rate in the real world is a percentage of worker salary and thus would presumable differ across worker skill-types as high-skill workers are generally paid more. Another assumption that may differ from reality is workers of both skill-types producing the same output in low-skill jobs. It's plausible that high-skill workers in low-skill jobs - underemployed workers - produce less than they would at a high-skill job but

more than low-skill workers. If this were the case, we would also see a third wage tier in the model with high-skill workers earning more than low-skill workers at low-skill jobs but not as much as their high-skill peers at high-skill jobs. In summary, there are many factors that could explain the lack of correlation between the model's prediction and observations within the data. While the exact reason is not uncovered here, it is likely do be due in some way to the one-period assumption along with the other factors discussed.

4 Conclusion

This paper explores the relationship between underemployment and unemployment insurance within a general-equilibrium framework. To predict underemployment rates within the college-educated workforce, a one-period model is constructed. The model exhibits heterogeneity across both worker- and skill-types. Firms endogenously post a mix of low- and high-skill job offerings, after which workers undergo an undirected search process. This results in an equilibrium in which workers are either unemployed, employed, or underemployed in the case of high-skill workers matching with low-skill vacancies. Underemployment exists in the model due to increases in the replacement rate driving the surplus created by low-skill job matches relatively lower than those that are high-skill.

To measure skill-mismatches within the workforce, differences between job educational requirements and worker educational attainment are used. Individual-level data is gathered to inform demographic and educational information and then aggregated at the state level. The principle findings, under a regression based analysis, are that when controlling for state and year fixed effects,

unemployment increases with the replacement rate - not in a highly significant way - while underemployment is decreasing with the replacement rate. In observing the effects of controlling for gender and marriage groups on underemployment, we see that the most responsive group to changes in underemployment is married men followed by single females. While outside the scope of this paper, this division is an area of interest. There are no immediate answers as to why this may be the case, or to why the significance in marital status flips across gender groups. These results are not significant to the 1 percent level leaving the door open for Type I error.

In weighing the model's qualitative predictions against the data, the model holds up well. In both the model and data, unemployment increases with the replacement whereas underemployment decreases. With regards to workforce education, both unemployment and underemployment are decreasing. However, in calibrating and parameterizing the model to judge the magnitudes of these predictions, the model's predictions vary significantly from the data. The calibration strategy is reliant both on outside literature for some parameters and the observed data for others. Worker bargaining power and elasticity of matching with respect to vacancies are fixed while the rest of the parameters are allowed to fluctuate across states over time. This allows for the simulation and reproduction of the observed historical period for both unemployment and underemployment. In judging the predicted values against the observed, there is no evidence to suggest that the model's predictions are correlated with reality. There are multiple potential explanations for the lack of correlation, including the one-period foundation and other abstractions from the real world.

However, despite the model's lack of predictive capability, this paper lays a basis for how a

general equilibrium framework can be used to model macroeconomic trends. The model is also not without merit, as seen in the qualitative predictive capabilities. On the quantitative front though, more work is needed to refine the model. This likely includes the introduction of a parameter for matching efficiency which could be calibrated in such a way to lower unemployment rates to better match the data. Underemployment is naturally hard to model. Unlike unemployment, it does not follow general business cycle fluctuations and is significantly harder to measure. However, it is still an area worthy of study and a perfect application for the development of heterogeneous search models.

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