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Epistemic Classism in Elite College Mathematics Education

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Abstract

This project started with my own experience of a shift in mathematical language and epistemology once I enrolled at an elite college. I wanted to explore how much of this shift was due to social class dynamics, how much epistemic injustice (injustice as it relates to knowledge systems) was occurring. Thus, my focus on epistemic classism in elite college mathematics education. I observed a single introductory calculus class for first and second-year undergraduate students at an elite college, conducted three rounds of interviews throughout the semester with ten students, interviewed the professor, and took some field notes during class time.

In line with Bernstein's (1964) work on social class and language, I found that there were codes of mathematical language that differed by social class, with more upper-class students fluent in *elaborated mathematical code*. More working-class students had to learn this language throughout the semester, as they were surprised at how abstract the class was. Working-class students saw math as not valuable, which can be viewed through Anyon's (1981) themes of *resistance*. Meanwhile, upper-class students saw math as a way of conceptualizing the world. Finally, I found that math has a special status in society as compared to other disciplines. It is regarded as a difficult area of study, making grades especially important to showcase status. This was internalized by all students, while 'math is for everyone' was a strictly upper-class belief.

Lastly, I framed my findings using the philosophy of epistemic virtues. I found that the elite values math and reason in-and-of itself, and thus have a *self-justifying epistemic virtue of rationality* which has high social status. It was imposed on students of all social classes by the class and the elite institution of the college. More working-class students had an epistemic virtue of rationality that was not as self-justifying. This imposition, this ignored mathematical epistemological mismatch between the college and its students, is epistemic classism.

Introduction

My first memory of being seen as "good" at math was in my play-based kindergarten class. As a tradition during the moving-on ceremony, the students would take turns counting as high as they possibly could. When it was my turn, I kept counting and counting until the teachers realized that I was going to continue counting until I reached the thousands; so, they had to stop me. Later on, in my multi-age public elementary school classroom, where two or more grade levels often inhabited the same space, I remember solving a particularly difficult math word problem and being proud enough to tell my parents about it. I remember scoring well over two hundred division facts in five minutes. I was seen as the "math kid" my entire public school career, skipping enough math classes to run out of them by my junior year of high school. My role in mathematics classrooms was to teach my tablemates. I understood the language.

At the same time, I couldn't stop thinking about what jobs used math in ways that I liked. There weren't a lot of them. Nevertheless, throughout my middle school and high school years, a constant was my mother telling me to never become a public school teacher. My entire family is littered with teachers, and my mom teaches at the public elementary school in my hometown. She was even my third-grade teacher in a mixed-aged classroom, Mrs. Olsen. While she loves her job, loves education, and always believed I could be an inspired teacher, a part of her didn't want her child to enter a profession where she, just like teachers nationwide, had been overworked, underpaid, and underappreciated by American society. Like every parent, she wanted better for her children, whatever "better" means.

Once I started thinking about my future, however, I couldn't imagine myself working for a profit-driven company. I wanted to fight the good fight and let what privilege I have be used to help others who deserve it. College mathematics professor was pretty much the only path I could see for myself, until I realized that there's a reason that pure mathematics professors tend to be the weirdest people on campus—you have to be crazy to go to graduate school for abstract mathematics for six years. I simply wasn't crazy enough. Granted, I was warned by my math professor during my freshman year that many pure mathematics majors burn out by the end of their undergraduate studies. Although, I like to tell myself that I didn't exactly burn out. Because as much as I cherish the language of mathematics, the pure math path I was headed down felt removed from social change, removed from people. After all, if there's any meaning for me in this world, it comes from the love between people and for people.

I found that love in introductory education classes during my junior year. At the root of the discipline is a care for people, for a community. Finding a community to be a part of for the long-term, creating lasting connections with people and places, and assisting change for the better are all that I want. Perhaps that's short-sighted. Perhaps I could improve the quality of life for millions through work in applied math and computer science. Perhaps teaching, for me, is a selfish choice of job in some sense. But it's who I am through nature and nurture. And it's been who I am for longer than I thought. Weeks into my freshman year of high school, I realized that school was merely a game to be played. The memorization, the rote learning of most classes, and the literal points-based output sat wrong with me on some level. I made a contract with myself that night in 2014. It read: "If I win the game I choose to play I will advocate against it." I still have the signed paper. At the time, I was merely thinking of the speech I got to give at graduation based on my GPA. But looking back, 9th-grade me was astonishingly prescient in predicting my life passion. I had to let my mom down.

I say so with a tinge of sarcasm, of course, but the wisecracking banter of teachers wondering why they ever chose to teach reflects the realities of the career I'm excited to foray into. Teaching is one of the hardest jobs one can do for the money, especially at an underfunded public school. I mention this to emphasize that this research was not undertaken to criticize teachers for the sake of criticizing teachers. Yes, there are ways that every teacher can become more equitable and more effective in their practice, but I wanted to undertake an analysis of education using a framework of systems first and foremost.

I found a new system at my little ivy college, one of social class at Colby College. Coming from a middle-class background, the elite culture of both the student body and the institution of my college created a shock to my system that I didn't fully understand at the time. Back at my high school, nobody remembered the last person that went to an Ivy League or little ivy school, and hell if my friends had ever heard of the colleges I was applying to. I didn't even really know the landscape of colleges, and looking back, I should have been more ambitious with my applications. But it was a new world. Colby was a new world. I even flirted with considering transferring to one of my state's small public colleges out of a feeling of betrayal of my community. Despite all their faults, I felt a kinship with the people in my high school that I couldn't shake. A year or two later, education courses started to address that deep-seated discomfort that I had at Colby but couldn't articulate. It was about social class. At my small public school in a middle-class farm town, even though classism was commonplace, there was a far more representative distribution of social class in the student population. Once I got to Colby College, the disgusting lack of representative socioeconomic diversity and jaw-dropping prevalence of students from elite families created an environment where I felt a little bit out of place, at least as much as a white, cis, het, able-bodied male can truly feel out of place in such a situation. I became engrossed in learning about how our country's education systems reinforce social class, whether it be through classed tracking, the myth of meritocracy, funding, or implicit biases in educational systems. I then found my intersection, a little nook that I want to explore—education, mathematics, and social class.

That brings us back to my experience with mathematics. Predictably, I was in for a doozy with mathematics at Colby College. Freshman year was an intensive proof-based two-semester sequence for prospective mathematics majors. Over sixty people were there on day one; only a dozen or so finished the sequence, including myself. I was in a new world of abstraction, learning a new language guided by a textbook that my high school would never dream of using. It was immensely rewarding. But it was remarkably foreign. When I tried to explain my mathematics classes to my family and friends back home, I struggled to convey exactly what we did. Even my family of public school teachers was perplexed as to how foreign my mathematics classes were. My experience was admittedly unique, as only a handful of the students at my school took that sequence. I still wondered though, how much of that class's language was a result of the mathematical proof-based nature of the sequence versus how much was a result of the elite academic environment of a little ivy? And how do those two potential causes interact with each other? These initial questions and ideas gave birth to an exploration into epistemology and social class, forming the theoretical background for this research, as well as birthing questions about what it means to talk about mathematical language and epistemology.

After all, whether it be Creole, Hindi, Spanish, the language of mathematics, the language of anthropology, or the language of elite institutions, language and epistemology are deeply intertwined. In America, bell hooks (1994) writes about how black vernacular speech "forges a space for … alternative epistemologies—different ways of thinking and knowing." Mathematics has shaped my epistemology, just as any other language does. When traversing through other academic disciplines, I find my thinking is often influenced by the mathematical language, the mathematical epistemology that I have habituated into over the years. For example, it has imbued within me a sense of positivism, a sense that still rests within me to this day, even if I can now better recognize its limitations. It has become part of me, and I cherish it. However, such a value judgment will be different in different academic subjects, different contexts, and different cultures. The same can be said about any particular language and any associated epistemology that creates and is created by that language.

Historically, when colonialism destroys cultures around the world, traditions and lives aren't the only things that are taken. That culture's epistemology is also destroyed. Their way of comprehending, understanding, and knowing the world is erased. In the late 19th century, ethnomusicologists in America (also known as comparative musicologists) realized that Indian cultures were being erased, and if they didn't capture the sounds of their music, it would be lost to time (Nettl, 2005). So, they did extensive fieldwork and tucked the recordings away at elite white male institutions. Only decades later did they give true access to these recordings to the people they recorded. They are scratchy and hard to make out, yet they are massively influential due to the glimmers of their culture's epistemology that they contain (Nettl, 2005). In those words and sounds, are new old ways of understanding. This goes to show that although a culture's epistemology will naturally change over time as the culture changes, the phrase "epistemic injustice" can serve as a fruitful lens to understand the far-reaching, almost spiritual impact that violence against a people, physical and otherwise, can have.

In many mathematics classes, there is an adherence to a certain mathematical epistemology and an emphasis on cultivating a respective virtue of rationality. Those emphasized virtues and epistemologies vary across cultures. In some ways, as I have noted, they help create culture. They can include varied emphases on the credibility of empirical data, the reliance on deductive logic, and the threshold for what constitutes knowledge. Now, there is nothing inherently wrong with emphasizing a particular way of understanding the world within certain academic disciplines. There is nothing wrong *per se* with acknowledging that certain people have epistemic authority in a given epistemology and culture. But, there is potential epistemic injustice when that particular way of understanding is 1) enforced in a way that seeks to undermine ways that other groups of people understand the world, and 2) puts groups of people who aren't from privileged backgrounds at an unreasonable disadvantage. Lower-income and working-class students face a massive shift in the culture, language, and epistemology of their peers and institution when they attend an elite college. Such epistemic injustices have been theorized to exist in mathematics classrooms by the small emerging field of the philosophy of mathematical practices, a field directed at math educators (Tanswell & Rittberg, 2020). Yet, there isn't any research directed specifically at how these epistemic injustices manifest in the classroom.

That then will be the main goal of this research project. I hope to illuminate how students from different social class backgrows interpret the language and epistemology of mathematics in an elite educational setting. What constitutes mathematics knowledge for students from social class? How does the language of mathematics differ across social class? And in what ways do student attitudes towards mathematics and social class background correlate? In answering these questions, I plan to encounter themes of epistemic classism, status, and the ways they are reinforced in a classroom setting.

Literature Review

Social Class and Higher Education

Much of this section can be framed as part of the motivation for this study, perhaps as a more academic extension of the introduction. Over the past fifty years in America, the disparity in household income between upper-class and middle/lower-class households has been steadily rising, as has the disparity in household wealth (Horowitz et al., 2020). Although this rising economic inequality is rather well-known, less known is the increasing public school-based self-segregation by social class in America. Segregation by household income between school districts increased by 15% from 1990 to 2010, and the segregation by social class between schools within districts increased by 40% from 1990 to 2010, as measured by the number of students eligible for free and reduced lunch (Owens et al., 2016). Neighborhoods and public schools are becoming increasingly insular with regard to social class, making colleges places where disparities in social class take center stage in day-to-day interactions and processes. In the 21st century, as capital is increasingly concentrated in the hands of the few, the sociological study of elite institutions has become more important and more popular (Khan, 2012).

It doesn't help that elite colleges are some of the least socioeconomically diverse institutions of higher education. At elite colleges, the roughly eighty most selective colleges and universities in the United States, the share of students from the top 1% has climbed steadily from below 10% to 11.5% from 2002 to 2013 (Aisch, et. al., 2017) even as society claims to be fairer and more just. This causes immense stress for low-income and working-class students at Colby College and other similar colleges, as they have to worry about expenses related to travel, cars, social activities, family visits, laundry, textbooks, and various other things (Jack, 2019) . The expenses of social activities are especially important, as they create a system where only upper-class students can participate in going out to dinner, concerts, skiing, or day trips around the area. It devastates low-income and working-class students' sense of belonging (Aires & Seider, 2005).

But the inequities go beyond economic capital. This study will focus more on social and cultural capital, particularly the latter. Social capital is the network of personal relationships that an individual has, whether that be through friendships or family, and the goodwill and advantages that those relationships can bring. It is not, however, merely about having a large social network. Social capital is accrued when one has a social network with people of the right people and the social status to take advantage of those connections (Bourdieu, 1987). As an illustrative example, one of my friends at college recently told me that over the summer he was going to be mentored by a member of the upper management for a large financial company. I asked how he got that job, he just said: "through his dad." Note that social capital also requires social status and knowledge to effectively access social network resources. This becomes especially important at college, where students are introduced to a fresh social network of both peers and adults.

Cultural capital, on the other hand, is less concrete than economic or social capital. It is the set of material and symbolic goods that an individual acquires when part of a particular social class, including education, mannerisms, style of speech and dress, skills, tastes, material possessions, credentials, etc. Cultural capital is the way to operationalize the phrase "people like us" when it is referring to social class. Beyond this, it is important to recognize that certain cultural capital is deemed more valuable by the dominant culture and institutions. This, in turn, can hinder or help a person's social class mobility just as much as economic capital or social capital. In Bourdieu's theory, habitus is the embodiment of cultural capital, comprising skills, dispositions, and habits that we have due to life experiences (Bourdieu, 1987). It creates a “feel for the game.” Upper-class students already come in with a habitus that is effective within the elite culture and institutionalized values of elite colleges.

There are important distinctions to make, however, within the ranks of low-income and working-class students at elite colleges, as they are often treated as a homogeneous group. One of the most important is the one Anthony Jack (2019) makes between the privileged poor and the doubly disadvantaged. The doubly disadvantaged are students from low-income and working-class backgrounds that attended their neighborhood public school. The privileged poor are students from low-income and working-class backgrounds that attended elite private or alternative high schools that imbued them with some of the social and cultural capital of the upper class. They are more familiar with the unspoken rules and expectations of an elite (educational) institution, such as what office hours are and how to take advantage of institutional resources. For example, the privileged poor are more at ease with authority professors and are primed to engage with them. They share some of that cultural capital and habitus with middle, upper-middle, and upper-class students. The doubly disadvantaged, on the other hand, tend to resist engaging with authority figures at college (Jack, 2019). Of course, both groups share many experiences, such as struggling through closed dining halls during breaks, finding transportation, needing campus jobs, and being forced to perform their poverty in a variety of situations (Jack, 2019). Due to the perverted nature of admissions at elite colleges, none of the participants in this study are true members of the doubly disadvantaged.

So, how does social class manifest in the setting of an elite college beyond the examples I have given? First, I will answer this from the standpoint of graduation rates and grades. At American colleges and universities (not necessarily elite ones), low-income and working-class students are around 10% less likely to graduate compared to middle, upper-middle, and upper-class students (Espenshade and Radford, 2019). Moreover, 36% of low-income and working-class students graduate in the bottom 20% of class rank and only 13% graduate in the top 20% of class rank. For upper-middle and upper-class students, only 15% graduate in the bottom 20% of class rank, while 25% graduate in the top 20% of class rank (Espenshade and Radford, 2009). Either we accept that low-income and working-class students have lower average intelligence than upper-middle and upper-class students, or we accept that higher education systematically and unfairly disadvantages lower-income and working-class students.

More sinister yet is the effect social class has on students' college experiences beyond the easily measurable outcomes of graduation and class rank. Aires and Seider (2005) interviewed 30 lower-income students, 15 from a little ivy and 15 from a State College, they detailed the common themes of the impacts of the lack of economic, social, and cultural capital that lower-income students experienced. At the little ivy, chasms of capital resulted in inferiority, exclusion, and powerlessness. Feelings of inferiority came from their newfound inability to articulate their ideas as clearly, due to the alien language of both their affluent schoolmates and the little ivy itself. This supports Bernstein's (1964) theory of language, with different social classes using distinct codes of language. Moreover, feelings of inferiority came from newfound deficit mindsets regarding their parents and upbringing, as they became aware of the supposedly "culturally richer" childhoods their schoolmates had. These feelings were most prevalent among first-generation students. Next, feelings of exclusion came particularly in the form of social exclusion and were not concentrated in first-generation students. All lower-income students were excluded from the social activities of the more affluent students due both to having different tastes and preferences as well as affluent student lifestyles that required resources such as disposable money. Lastly, feelings of powerlessness came from the realization of the difficulty to alter the feelings of inferiority and exclusion, as well as relative powerlessness to shape their future through expensive graduate schools and other elite career paths, as they lacked both the economic and social capital to do so. At the state school, none of these themes were nearly as pronounced, as the students interviewed were not constantly surrounded by affluent students and elite institutional norms.

Still, social class identity and learner identities are not one and the same. They overlap, interact, and shape each other, and they can be used as distinct social identities. For example, Diane Reay (2009) discusses how even before attending elite colleges, working-class students in the United Kingdom are often,

> faced [with] the paradoxical situation of being more like a 'fish out of water' in their largely working-class state secondary schools where their highly developed academic dispositions fitted uneasily into the field of predominantly working-class schooling. The irony is that they have a greater sense of fitting in as learners in elite higher education than they had at school surrounded by 'people like them' … (p. 1115)

Of course, the phrase "highly developed" is normative and comes from the dominant upper-class culture of what it means to have a good academic disposition. Nevertheless, it is important to recognize the differences between social class identity and learner identity. And when working-class students don't share the learning identities enforced by the institutional habitus of the college they attend, the ideas of what it means to be educated and what it means to be working class are often put at odds, resulting in parents that want their child to attend college but are afraid their child will abandon what it means to be working-class (Reay, 2010).

Lastly, although I will not specifically focus on it in this study, the disparate experiences that students of different social classes have of the social life and extracurriculars at college deserve to be noted. Jenny Stuber (2011) has done extensive work in this area, demonstrating the refined maps and navigational tools (i.e. habitus) that middle/upper-class students have for traversing the social and extracurricular worlds of college. This only exacerbates the gap in social capital and cultural capital that students have. The case of mathematical knowledge, however, is more subtle. While students from marginalized social classes face all of the aforementioned obstacles at elite colleges, they also face a challenge related to cultural capital—epistemology.

Epistemology

In the simplest of terms, epistemology is the study of knowledge. This encompasses theories of knowledge acquisition, validity, scope, and whatever other methods are relevant to knowledge systems. Knowledge is colloquially described as a justified, true belief. This project will use the term epistemology in a broad sense, encompassing all facets of knowledge systems. Hence, a decent place to start our discussion of epistemology is with a question. What is the most relevant unit of study in epistemology? Traditional Western epistemological theories are typically foundationalist, as seen in Descartes's Meditations (Descartes and Cress, 1993), as well as various forms of coherence theories (Sosa et al. 2008). Foundationalism is the view that all knowledge needs to ultimately rest upon a foundation of non-inferential knowledge. This theory can be visualized as a pyramid, with knowledge following inferentially from previous knowledge in a linear fashion. Coherence theories can be visualized as a web of knowledge, where a proposition is accepted as knowledge when it fits into the web of knowledge that a person already has. They are both convincing metaphors. There have even been attempts to blend the two approaches, forming a sort of "foundherentism" (Haack et al., 1994). Nevertheless, while there are vast variations in all of these approaches, with wrinkles and nuances abound, they all share a focus on the proposition as the relevant unit of epistemology. When debating whether something "counts" as knowledge in these more traditional theories, the proposition itself is what matters. This framework of epistemological study stands in close relation to the mindsets of analytic philosophy and positivism, both of which I have been heavily influenced by as an epistemic agent.

These traditional theories stand in stark contrast to progressive models of education, where teachers often emphasize that they teach students, not topics. Undoubtedly, the most relevant unit of epistemology in progressive theories of education is the student. People who subscribe to the ideas of philosophers of education such as John Dewey, Paulo Freire, and many others describe these progressive models in different ways, including critical inquiry vs. banking models, 'the method in the message,' and 'how you teach is what you teach.' While educators rarely think in terms of inculcating explicit virtues, their goals for teaching lie in creating learners (in epistemological lingo, knowers) that are open-minded, enthusiastic, responsible, or whatever set of epistemic virtues are valued in that community's culture. In epistemology, Linda Zagzebski's pioneering book *Virtues of the Mind* (1996) argued for a *virtue epistemology* that changes the epistemological agenda in a way that many educators and philosophers of education would find familiar, through shifting the focus from the proposition to the knower. Specifically, she argues for analyzing the intellectual virtues and vices of the knower as the best way to understand knowledge. If at this juncture the reader is concerned that I am straying from some concrete, indisputably true form of knowledge, one only needs to read Descartes and his fellow skeptics to realize that such a goal is impossible. Skepticism reveals such significant cracks in foundationalism that we need to explore new theories, new metaphors. We can do that with virtue epistemology.

First, what are virtues and what are vices? Such ideas are normative and emerge from the field of ethics. A subset of this field, virtue ethics is the theory that morality comes from virtues and vices, which are cultivated dispositions towards how one acts in the world (Hursthouse, 2018). These virtues and vices are not habits, they are character traits core to one's personality and identity. Virtue ethics originated in ancient Greek philosophy, and the thinkers of that time considered virtues and vices to be both intellectual and affective (Annas, 1993). That is, it was important to be able to reason through a situation and make a good ethical decision as well as to apply your emotions correctly to that situation. We still carry that definition of virtues and vices, although determining exactly what is a virtue and what is a vice is markedly more normative.

For illustration, what do some of these epistemic virtues look like? Aristotle listed some of his intellectual virtues in his *Nicomachean Ethics* (Aristotle, 2019) including:

1. Nous (intelligence) – apprehension of self-evident truths
2. Episteme (science) – skilled inferential reasoning
3. Sophia (theoretical wisdom) – combining fundamental truths with inferential reasoning to realize more unchanging truths

Notably, Aristotle lists only these three as intellectual virtues. Modern virtue epistemologists would likely consider many of the other virtues that he lists as epistemic virtues, such as practical wisdom (the knowledge of changing truths). Even John Dewey (1916) had a far more expansive set of intellectual virtues long before the idea of epistemic virtues, as he included open-mindedness, responsibility, and whole-heartedness as intellectual virtues. These more nuanced and interpretive epistemic virtues are what I am interested in, especially as I get into specifically mathematical epistemic virtues later on.

For now, let us stay within general epistemology. As the Enlightenment came over Europe, virtue ethics fell out of favor as consequentialist ethics and deontological ethics fell into favor. Then, in the second half of the 20th century, in parallel with the rise of postmodernism during this time, virtue ethics started to become more fashionable again. Just as postmodernism framed knowledge and value systems as socially-constructed and highly contingent, the virtues and vices in virtue ethics could be framed as socially-constructed and contingent on the communities they exist in. After all, inherent in any epistemology that isn't reducible to human psychology is normativity (Kornblinth, 1995).

It was in this environment, at the very end of the 20th century, that Linda Zagzebski's book *Virtues of the Mind* (1996) was published. It is easy to understand how different cultures may have different ethical virtues and vices, as every culture has varied pasts and experiences that give rise to varied ethical virtues and vices. Yet, Zagzebski (1996) creates a clear distinction between context-sensitive and environment-neutral epistemic virtues. The most striking part of this distinction is in its assertion of the existence of environment-neutral virtues. Yet, Zagzebski also notes that "the path to knowledge varies with the context, subject matter, and the way a community makes a division of intellectual labor" (p. 177). Using that logic, doesn't a different path imply a different destination? Doesn't it imply different, yet equally valid, epistemic virtues? I would argue as much. We can never call a virtue environment-neutral, epistemic or otherwise. Even valuing reasoning itself is relative to the context of the human experience, never mind the ways the concept of reasoning may function differently across languages, across places, and crucially for this study, across social class. Many epistemic virtues and vices may be shared across cultures, but they cannot be taken for granted as universal.

In education, Zagzebski's (1996) virtue epistemology allows us to understand teachers as inculcating culturally sustaining, context-dependent, environment-dependent epistemic virtues in their students through culturally relevant/sustaining pedagogy. Different communities and thus different public schools within America can "teach" or "inculcate" different virtues and vices. This is an essential part of understanding pedagogy as focusing not only on the student but on the norms and values of the community, including (crucially) epistemic ones. Education is not a process undertaken objectively through identical curricula, pedagogy, hierarchies, and practices. Instead, education is necessarily relevant to a community, whether that community is a small farming town in the early 1800s or the state of Texas in the 21st century. Even as globalization has expanded and altered the boundaries of community beyond our neighborhoods, students are sent to charter schools, magnet schools, and private schools whose community norms and values parents feel more aligned with. Thus, even as other schools have started to challenge the neighborhood public school, community norms and values, including epistemic virtues and vices, still reign supreme. Virtue epistemology is an extremely fruitful framework.

For now, I have set up the scaffolding to explore and define a primary concern of this project—epistemic injustice committed against the oppressed. First conceived by Miranda Fricker and popularized by her 2007 book *Epistemic Injustice: Power and the Ethics of Knowing*, epistemic injustice is just what it sounds like; injustice related to knowledge systems. Although it is still a mere subsection in pages dedicated to virtue epistemology and social epistemology in *Stanford Encyclopedia of Philosophy*, the idea of epistemic virtue/vice of epistemic justice/injustice is an increasingly fashionable tool to frame systems of injustice more profoundly. Below is a handy table that outlines Fricker's (2007) conditions for epistemic injustice, formatted by Byskov (2021). These are not the be-all and end-all of epistemic injustice, as we shall soon see with Hookway (2010), but it is a valuable overview.

Table 1

Fricker's Conditions for Epistemic Injustice

Condition	Description	Aspect of (in)justice
The disadvantage condition	In order for someone to be unjustifiably discriminated against as a knower, they must suffer epistemic and/or socioeconomic disadvantages and inequalities as a result from the discrimination.	Unfair outcome
The prejudice condition	In order for someone to be unjustifiably discriminated against as a knower, the discrimination must involve prejudiced (i.e., unfair) sentiments about the speaker.	Unfair judgment about epistemic capacity
The stakeholder condition	In order for someone to be unjustifiably discriminated against as a knower, they must be somehow affected by the decisions that they are excluded from influencing.	Unfair denial of stakeholder rights
The epistemic condition	In order for someone to be unjustifiably discriminated against as a knower, they must possess knowledge that is relevant for the decision that they are excluded from.	Unfair denial of knowledge
The social justice condition	In order for someone to be unjustifiably discriminated against as a knower, they must at the same time also suffer from other social injustices.	Unfair existing vulnerability

More importantly, Fricker (2007) outlines two distinct types of epistemic injustice, *testimonial injustice* and *hermeneutical injustice*. Testimonial injustice occurs when someone's claims are given more or less credence as a result of prejudice. These prejudices could be based on race, sex, age, ethnicity, culture, gender, or any other form of identity. Hermeneutical injustice occurs when people of a certain identity are denied or stripped of the epistemologies and conceptual frameworks/language needed to understand their world and to communicate.

In hermeneutical injustice, we explicitly encounter the deep-seated importance of language and other ways of communication in understanding epistemology. Conceptualizing in a broad sense to include the nuances of communication, language and epistemology truly create each other. When people are colonized and their languages and customs forcibly changed by the colonizer, their culture is lost. That is the most common lament. But also lost is the epistemology of that culture, the way that they understood the world around them. Epistemic virtues/vices that result in hermeneutical epistemic injustice are so insidious because they forcibly destroy the systems of epistemic virtues and vices and other communities. Both testimonial and hermeneutical injustice are difficult to combat, and others have emphasized the necessity of dialogical communities to expose and confront them (Alfano, 2015; Medina, 2011). I hope this project supports such a dialogical community.

Hookway (2010) expanded on Fricker's relatively narrow conception of epistemic injustice in ways that I will use implicitly throughout this project. While Fricker was concerned with injustices that are distinctly and solely epistemic, Hookway is also interested in injustices around epistemology that arise from circumstances that are less directly epistemic. For example, he gives an example of a teacher who happily provides students with information but fails to take their students' questions seriously. In doing so, the teacher strips their students of the ability to think of themselves as active participants in knowledge creation and acquisition. The teacher is refraining from inculcating epistemic virtues of curiosity and responsibility. No assertions are being made by the students, as Fricker would require, for testimonial injustice, but there is undoubtedly epistemic injustice committed, as the teacher neglected to recognize the student as an epistemic participant (Hookway, 2010). In this way, Hookway is broadening the scope of what is considered an epistemic injustice. While he does not consider the inequitable distribution of resources that result in knowledge as a distinctly epistemic injustice, such as the funding of school districts, it seems to me that even those situations contain a distinct epistemic flavor, an epistemic consequence. I would consider such injustices at least partially epistemic.

Enough theory, time for a real-world example. Virtue epistemology and epistemic injustice can be used as a lens not only to understand teachers but to understand larger education policy and institutions. Starting in the Reagan era with the 1983 report *A Nation at Risk* from the Department of Education (National, 1983), "excellence" has been the main driver of education policy in America of the late 20th and 21st centuries. That is, excellence as measured by scores on standardized tests. As a result of the traditional nature of the majority of standardized tests that students currently take and have taken, this "excellence" movement devalues certain epistemic virtues. It only values what they know from the perspective of the banking model of education, as that is all typical standardized tests measure. The injustice occurs when this "excellence" is enforced inequitably across schools. Historically marginalized neighborhoods and school systems that struggle to meet state standards have even less choice in choosing their epistemic virtues, as if they don't put all of their energy toward the banking model of education and raising test scores, they are likely to face sanctions, state takeovers, and school closures.

Plus, high test scores are often the only pathway towards social class mobility. Meanwhile, more affluent schools that already have acceptable test scores are free to choose their own epistemic virtues. In this way, the federal and state governments committed epistemic injustice by imposing an epistemology devoid of certain intellectual virtues (such as those outlined by Dewey) on school districts that struggle to meet standards that were passed by those who graduated from schools whose teachers had the freedom to inculcate an epistemology full of the virtues of their choosing. American education is suffocated by the myth of meritocracy, and it should then be no surprise that progressive education is historically a rich, white movement (Delpit, 2006). Virtue epistemology and epistemic injustice are invaluable frameworks for understanding this type of oppression in education.

I must end by stressing that I am not asserting the primality of any epistemology. Are certain epistemic virtues better suited to different contexts and cultures? Of course. But the epistemology of every culture (and thus the epistemic virtues of every culture) provide a valuable and unique way of understanding how things hang together in our world. We must not deny peoples access to those epistemologies, either by exclusion or destruction. And we must not ignore these differences, paving them over in the process. We must not commit epistemic injustice.

Social Class, Language, and Epistemology

People of different social identities often share different languages and different epistemologies. While the first examples that come to mind might include ethnicity and religion, the second half of the 20th century gave us research about social class, language, pedagogy, and thus epistemology. Crucially, Weinberg et al. (2001) argued and provided evidence that epistemic intuitions vary across social class. This variance is important to tease out and be cognizant of, as disciplines are exploring how confronting epistemic marginalization is integral to confronting inequity at large (Julian, 2017). In particular, Basil Bernstein's (1964) work in language codes and Jean Anyon's (1981) work in school knowledge provide fantastic foundations to conduct such research in the world of education.

Nevertheless, I will first start with the work done in *'Normativity and Epistemic Intuitions'* by Weinberg et al. (2001) for motivational purposes. Their work preliminarily showed epistemic intuitions varied significantly with social class. They presented participants with a pair of stories in which it was debatable whether the character in the story really knew something or only believed it. An example of one of these stories is shown below:

Table 2

Question From 'Normativity and Epistemic Intuitions'

It's clear that smoking cigarettes increases the likelihood of getting cancer. However, there is now a great deal of evidence that just using nicotine by itself without smoking (for instance, by taking a nicotine pill) does not increase the likelihood of getting cancer. Jim knows about this evidence and as a result, he believes that using nicotine does not increase the likelihood of getting cancer. It is possible that the tobacco companies dishonestly made up and publicized this evidence that using nicotine does not increase the likelihood of cancer, and that the evidence is really false and misleading. Now, the tobacco companies did not actually make up this revidence, but Jim is not aware of this fact. Does Jim really know that using nicotine doesn't increase the likelihood of getting cancer, or does he only believe it?	
Really Knows	**Only Believes**

In this situation, participants of low social class were split fifty-fifty between "really knows" and "only believes," while only ~18% of high social class participants chose "really knows" and ~82% chose "only believes." There was a similar discrepancy in an additional question of the same form, where ~33% of low social class participants chose "really knows" compared to ~11% of high social class participants (Weinburg et al, 2001). What could the cause of these differences be? Weinberg and colleagues posited that perhaps low social class participants considered possible but nonfactual states of affairs less relevant than high social class participants. Or perhaps low social class participants merely had lower bars for what they considered knowledge. There are of course several uses of the word "knowledge" that could have impacted the findings, but the researchers controlled their experiment for such systemic differences in the interpretation of "really knows."

Admittedly, extensive replication of this experiment has not been conducted, and a more recent attempt found significantly less significant differences in epistemic intuition between social classes (Seyedsayamdost, 2015). Still, epistemic intuition as measured by this experiment is only one component of the larger systems of knowledge. For example, Topçu and Yılmaz-Tüzün (2010) found that elementary school students metacognition was related to social class, where metacognition is defined as the awareness students have of their cognitive strengths and weaknesses, of the cognitive resources they have access to, and of the ways they can regulate their engagement to change learning processes and products (Perry & Winne, 2006). There are myriad ways to ask about and explore knowledge systems.

One of these ways is to examine language systems. In bell hooks' *Where We Stand: Class Matters* (2000), she talks about her grandmother, who was low-income and could not read or write. She could, however, tell stories. When bell hooks was around her family, "conversation was not a place of meaningless chit chat. It is the place where everything must be learned—the site of all epistemology" (p. 15-16). That type of classed epistemology is supported by Bernstein's work in language codes. In his early work, he defined restricted code and elaborated code as distinct types of language codes (Bernstien, 1964). Restricted code is less formal and relies on common assumptions, experiences, and understanding among those in conversation. It is a more efficient way of communicating, although it requires the audience to read between the lines. Elaborated code is more formal and does not assume any prior knowledge or experience, instead spelling everything out so that anyone can understand precisely what is being said or written. As a result, an idea spoken or written with elaborated code could stand alone and be understood without losing significant meaning, whereas an idea spoken or written with restricted code could not. Bernstein (1964) argued that working-class people use restricted code, while middle and upper-class people use a combination of both codes. Of course, restricted code is found everywhere. But importantly, working-class people do not have access to elaborated code. They aren't fluent in it. Because of this, their epistemology is distinct as well. With a lack of elaborated code comes a different way of understanding knowledge, a way that might be described as less formal. Or less abstract. Or… something. This is not meant to create a hierarchy of language code, as one is not better than the other. But Bernstein (1962) even found quantitative differences in phrase length and pause interval. These differences and codes are important not just as access points, but as cultural positioning devices that help position people in larger social relationships and define a social hierarchy (Bernstein, 2008).

Next up is Jean Anyon's (1981) work. The impetus for work like hers is the observation that "economically marginalized students are more likely to be subject to rote-like or skill-and-drill instruction" (Gorski, 2013, p.112). Essentially, knowledge as a concept is treated vastly differently in schools based on social class. She interviewed students, teachers, staff, and administration, as well as looked at the curriculum that the teachers used. Anyon (1981) classified the schools she observed as working-class, middle-class, affluent professional, and executive elite schools. While I did not use the same four distinctions in my data collection and analysis, and did not treat them nearly as systematically, the four social classes she used she defined in as follows:

Table 3

Anyon's (1981) Social Class Categorization

Working Class	Most parents have blue-collar jobs, with the majority having semi-skilled or unskilled jobs. Annual income is typically in the lowest thirty percent of US families. There is little if any family wealth, perhaps a modest house and an old car. Most people have no more than a high school education.
Middle Class	Most parents have traditional middle-class jobs such as public school teachers, accountants, public employees, middle managers, small business owners, etc. Annual income is typically in the next forty percent of US families. Family wealth typically consists of a house and two cars. Most people have a bachelor's degree from a public institution.

Affluent Professional	Most parents have jobs such as doctors, engineers, lawyers, college professors, etc. Annual income is typically in the next twenty to twenty-five percent of US families. Family wealth typically consists of a nice house, perhaps a second house, nice cars, and significant savings. Most people have advanced degrees, often from elite institutions.
Executive Elite	Most parents (typically fathers) are typically top executives at large corporations. Involvement in politics is much more common than in other social classes. Annual income is in the top five to ten percent of US families. Family wealth is extensive and consists of multiple houses, cars, savings, stocks, bonds, etc. Most people have advanced degrees from elite institutions and also attended elite schools for K-12.

Anyon (1981) found significant differences in the epistemology of schools that served students of different social classes. Starting with working-class schools, she found that school personnel described the most appropriate knowledge for children in terms of facts and basic skills. They needed the basics, and to be kept busy. In the mathematics curriculum, students were asked to carry out procedures without an understanding of why the procedures worked or the purpose behind the procedures. There was little to no student decision-making allowed. When Anyon asked the students what knowledge is, most had difficulty understanding the question, and otherwise gave answers similar to their teachers. When asked where knowledge comes from, six said from teachers, seven didn't know, and the few others had answers including "the dictionary" and "the board of education." When asked if they could make knowledge, almost all students said no. In general, students responded to questions in terms of behavior and skills, with only one student using the word “mind.” *Resistance* was the dominant theme, as students both actively and passively resisted teachers’ attempts to implement the curriculum. Teachers noticed this as well, as they expressed that students often didn’t care. Knowledge in these schools was composed of fragmented facts and simple mechanical procedures.

In middle-class schools, Anyon (1981) found that school personnel described the most appropriate knowledge for children in terms of facts and understandings that come from textbooks, as such content is decided upon and made by experts. Moreover, that type of content is what is needed for college and daily life, and holds more legitimacy than knowledge that is created by oneself. In the mathematics curriculum, multiple processes were given as options for solving a problem, and the teacher often taught and checked for understanding by asking a student how they did the problem. In this, there was a recognition of decision-making not present in working-class classrooms. When Anyon asked the students what knowledge is, the answers were varied but were thematically similar, ranging from “remembering” to “smartness.” When asked where knowledge comes from, most students answered with what would be considered resources made by experts, such as from “libraries” or “books.” When asked if they could make knowledge, nine said no and eleven said yes, however, most “yes” responses were followed with questions that revealed that most of the students’ conception of making knowledge largely consisted of looking things up or listening to experts. *Possibility* was the dominant theme, as students and teachers thought that learning things from the right places and thinking the right way provides an opportunity to move up. Content knowledge was seen as exchangeable for grades, college, or a job.

In affluent professional schools, Anyon (1981) found that school personnel described the most appropriate knowledge for children in terms of individual discovery, creativity, and making sense of their experience. In the mathematics curriculum, knowledge came from discovery and direct experience. Students participated in open-ended activities where they created and dissected patterns, formulating questions for classmates to help answer. When Anyon asked the students what knowledge is, the answers varied but were thematically similar, ranging from “brains” to “the way you think.” The word “think” was common. They seemed to focus on the idea of creatively making sense of the world. When asked where knowledge comes from, answers were split between the sources found in the middle class and abstract ideas of “your own head.” When asked if they could make knowledge, sixteen children said yes and four said no. Moreover, many of those who answered yes responded to follow-up questions in ways that revealed a more active approach to knowledge acquisition/creation than in the middle-class. *Narcissism* was the dominant theme, as knowledge was a deeply personal endeavor to make sense of the world. It was taught that individuals, not groups, make knowledge and history. Still, knowledge was also framed as a powerful resource for doing social good, and social studies were the most “liberal.”

In executive elite schools, Anyon (1981) found that school personnel described the most appropriate knowledge for children in terms of intellectual processes such as problem-solving and reasoning. In the mathematics curriculum, the primary goal is the cultivation of mathematical reasoning. Classes involved questioning that forces students to manipulate hypothetical variables, focusing on logical student thinking to solve the problem. When Anyon asked the students what knowledge was, the answers focused on a perceived necessity to understand existing knowledge. There was less of a focus on independently making sense of the world as compared to the affluent professional school, and instead, some students mentioned ideas around knowledge such as understanding, wisdom, confidence, and subjectivity of knowledge. When asked where knowledge comes from, thirteen out of twenty students responded “tradition,” “past experience,” or “other people,” with only a few responding along the lines of one’s own mind. When asked if they could make knowledge, ten said yes and nine said no. In the follow-up questions for those who answered yes, half gave answers similar to the affluent professional students, and half said that “it depends,” making distinctions between types of knowledge. *Excellence* was the dominant theme, as rationality, rules of reasoning, and traditional academic rigor were stressed. Knowledge was not something that could be created at will. Instead, knowledge comes from understanding the “internal structure of things: the logic by which systems of numbers, words, or ideas are arranged and may be rearranged” (p. 31). The pressure to perform and to think well is immense.

With these four schools, in conjunction with Bernstein’s work and the aforementioned difference in epistemic intuition, we start to see that there is a significant difference in epistemology with regards to social class, especially when framed in the field of education. Even when a government mandates a common focus, such as Singapore’s Ministry of Education’s explicit emphasis on critical thinking, that concept is interpreted and actualized differently depending on social class (Lim & Apple, 2015). Exacerbating this is the fact that even when adjusting for test scores, teacher recommendations, and student/parent choices, social class has a large effect on how students are tracked both between classes and within classes. In our schooling system, low-income and working-class students are overrepresented in low-ability tracks and underrepresented in high-ability tracks (Oakes, 2005). Even when we investigate other learning contexts, such as raising a child in the home, there are significant social class differences. For example, societal institutions value children who are experienced in communicating in a particularly rational way with adults and negotiating with authority, something children learn through parenting styles more typical of the affluent and middle class (Lareau, 2011), although there are pitfalls such as creating a sense of entitlement and depriving children of developmentally valuable free play time.

When interacting with the dominant culture, language and epistemology become a significant form of habitus and/or cultural capital as described by Bourdieu (1987). Because of this, Anyon (1981) describes the phenomena she observed as class conflict, and Bernstein (2008) described the phenomena he observed as class positioning. For this paper, I will call it epistemic classism, as that term encompasses more than just status-based injustice. That term captures both the systemic and individual sides of the phenomena and frames it squarely as an injustice committed against a people. Yes, I use it much more broadly than Fricker (2007), but I am justified in using strong words to describe a significant phenomenon. It is inescapable that there will be injustices everywhere, I am just investigating one of them.

Mathematics

Okay. How does this all interact with mathematics? We saw in Anyon's (1981) observations how mathematics was approached differently in the K-12 level. But that was a different type of observation, looking at how schools differ in their mathematical epistemologies. I am interested in how students that come from those different schools experience the mathematics of an elite college. In general, as I have mentioned, mathematics seems to occupy a unique place in our minds. Philosophers envy and try to imitate the certainty of mathematical arguments, and the public acknowledges the unique status of mathematics through the use of phrases such as a "math person" (Buldt & Muller, 2008). Nevertheless, professional abstract mathematical activity is often misconceptualized as a precise reading of carefully worded axioms, definitions, and theorems. This hides the truth of mathematical activity and how new mathematics has been discovered throughout history (Lakatos, 1976). Indeed, this misconceptualization extends to the undergraduate classroom, and Lakatos's constructs of mathematical activity can be utilized to guide and understand mathematical activity in the undergraduate classroom (Larsen & Zandieh, 2008).

This seems to require intense sociological study of mathematical activity, but mathematics is underrepresented in the sociology of science (Buldt & Muller, 2008). There is some of this work being done in the field of the philosophy of mathematics education. Over the second half of the 20th century, academics have observed that society started moving away from the primary goal of creating more mathematicians and towards the primary goal of creating a more mathematically competent general public and workforce (Dowling, 1997). What exactly this means is up for debate, and the tie between mathematics between mathematical epistemology and "critical thinking skills," while widely accepted by academics and policymakers alike, is somewhat theoretical and tenuous (Dawkins, 2020). Still, the field philosophy of mathematics education has started to tackle issues around mathematical agency, dialogue, and social justice in mathematics education, and the field recognizes Lakatos (1976) as the starting point for the field (Ernest, 2016). The philosophy of mathematics education has made it clear that ethnomathematics and mathematics education have extensive overlap and connection (Ernst, 2016). This project lies in that overlap, looking at how mathematical epistemology is situated in a certain cultural, educational space.

I am further motivated by research that has shown that mathematical activity abides by certain sociomathematical norms. An extension of the social norms that govern classroom inquiry and dialogue, sociomathematical norms and values govern mathematical discussions and mathematical activity (Yackel & Cobb, 1996). These norms relate to the judgment of what counts as an acceptable mathematical explanation, as well as to the judgment of mathematical answers themselves, comparing their sophistication, efficiency, similarities, and differences. Moreover, the teacher plays a critical role in shaping these norms (Yackel and Cobb, 1966). Thus, they have some natural variance from classroom to classroom. Still, as one can infer from Anyon's (1981) work, they are likely to vary predictably with regards to social class. When we frame those norms and values as epistemic virtues (as seen in virtue epistemology), we can talk about mathematical epistemology. Thus, as we saw in the discussion of Anyon's (1981) work in the previous section of the literature review, mathematical epistemologies will vary by social class as well. We can retrace our steps, looking back at how students and school personnel from each social class answered questions related to knowledge and how mathematics curriculum functioned in those schools. This is powerful. Then beyond K-12, in higher education, mathematical proofs themselves have values and norms such as decontextualized reasoning and creating/accepting a priori truth, values that likely vary across social class. Moreover, mathematical epistemologies are different depending on the academic setting, as learning the language of math in math classes versus the language of math in physics classes are two related but distinct prospects (Redish & Kuo, 2015). Thus, I will focus squarely on a mathematics classroom in this project, framing many of the interviews around the class, even if the class is not extremely proof-intensive. While it is beneficial to emphasize certain ways of understanding the world within different disciplines, as every academic subject has a unique framework of understanding, we should not expect all students that enter an elite space to quickly and easily accept those norms and values without critique and support. We know that this is a potential problem, as students' epistemological frameworks affect their interpretations of lectures in advanced mathematics (Krupnik, et. al., 2018). Even more importantly,

> ...undergraduate interactions with proof are more completely understood as a function of institutional practices which foreground particular epistemological frameworks while obscuring others. It is argued that enabling students to access the academic proof procedure in the transition from pre-university to undergraduate mathematics is a question of fostering an epistemic fluency which allows them to recognize and engage in the process of creating and validating mathematical knowledge (Solomon, 2006, p. 373).

While we already know there is epistemic injustice within mathematics education (Tanswell & Rittberg, 2020), we need an ethnography at an elite institution. Remember, I am not attempting a pointed criticism of any specific teachers or curriculum. I am instead trying to spark discussion about justice in mathematics education. Because although it is sometimes given a special place within the academic disciplines, mathematical activity is built on values and norms just like any other discipline. When those sociomathematical norms at an elite institution interact with a student body from a variety of social class backgrounds, it creates epistemic injustice, epistemic classism. This is borne out when the epistemology (epistemic virtues, epistemic intuitions, metacognition, sociomathematical norms and values, etc.) of low-income and working-class students don't match that of the elite institution *and* this mismatch is ignored.

Methods

Context

This study was conducted at a “little ivy” college, a small, highly-selective four-year private liberal arts college located in the northeast of the United States. For the purpose of this study, I will refer to the school as Miller College. Like other little ivies, it has a large endowment and a wealthy student body. In 2014, 20% of its student body was from the top 1%, 76% of its student body was from the top 20%, and only 2% of its student body was from the bottom 20%, all of which were in the ten worst among sixty-five elite colleges included in the rankings (Aisch, et. al., 2017). Tuition and room and board costs total over $75,000 as of the 2021-2022 school year. Although Miller College commits to meeting 100% of a student’s financial need, only forty percent of the class of 2025 received financial aid from Miller, again showcasing the wealth of the student body. Many admitted low-income and working-class students attended specialized high schools, such as private high schools, charter schools, or selective high schools. We will see this pattern when discussing the participants in this study. When thinking about social class, Miller College is an undeniably elite institution. The vast impacts of this lack of socioeconomic diversity and elite culture were discussed in the first section of the literature review.

Furthermore, Miller College is a predominately white institution, with approximately 30% of its student body identifying as a student of color, although that number has been very slowly rising, with the incoming class of 2025 composed of 37% students of color. Approximately 10% of its student body are non-US citizens. Students come from all over the United States, with Massachusetts as the top sending state by a fairly wide margin. Massachusetts students typically come from the suburbs around Boston, and are monikered JOBs (just outside of Boston). Other top sending states include California and Connecticut. Along with the designation as a little ivy comes many of the accepted practices, such as legacy admissions policies and unofficial slots for graduates from highly prestigious private high schools. Public high school students at Miller often come from quasi-public high schools which require massive wealth to purchase a house in the district, essentially creating a tuition cost for attending. For illustrative purposes, although I grew up in Connecticut, I don't have any friends or connections from my hometown or surrounding towns that are currently enrolled at little ivies, as I went to a very average, small public high school.

The math department at Miller College seems typical for little ivies. In the 2021-2022 academic year it had nine tenure-track professors, four visiting professors, a lecturer/calculus coordinator, a research associate, and an administrative assistant. As with most college math departments, the majority of the professors are white males, especially the older, fully-tenured professors. The department occupies the second floor of an academic building, with study spots that groups of students often utilize. The statistics department has its own major and recently moved from inside the math department to another building. Then, within the math department, there are multiple pathways. There are two majors, mathematics and mathematical sciences. The mathematics major is the abstract mathematics pathway, with proof-based mathematics emphasized in courses such as real analysis and abstract algebra. The mathematical sciences major is the applied mathematics pathway, with applications emphasized in courses such as ordinary differential equations and statistics. There is also a mathematics minor.

That being said, the majority of students in single-variable calculus I observed were first and second-year students and will not be math majors of any kind. Instead, many of the first-year future math majors take a two-semester sequence of proof-based calculus. Around twenty first-year students take that sequence, while alternatively, there are currently four sections of single-variable calculus with about twenty-five students each. The students in single-variable calculus do not generally take it with a mathematics major in mind. If they decide they want to pursue a math major after they have completed single-variable calculus and multivariable calculus, there is a one-semester mathematical reasoning course to take. It is more common for mathematical science majors than for mathematics majors to take this pathway. Nevertheless, most students in the single-variable section I'm observing take the class for other reasons. These include the quantitative reasoning distribution requirement that Miller College has, and which single-variable calculus fulfills, as well as other major requirements, such as economics, physics, and chemistry major requirements. Many take this class their first semester of college just in case they want to major in one of those subjects that require a semester of calculus.

Moreover, the single-variable calculus and multivariable calculus sequence at Miller was undergoing significant changes for the year of my study. There was a new calculus coordinator, standardized homework assignments, and standardized tests. The department was making an active effort to pedagogically reform its calculus classes. Still, this didn't drastically alter my study, as I focused on language and epistemology, and older professors are often secure in their ways. Still, there was some effect, and I made notes of that in my discussion and analysis.

All of this means that the single-variable calculus I'm observing is the contact point the majority of students at Miller College have with the math department. It is also often one of the first four classes students take at Miller, usually during their first or second semester. That explains the attention that the math department has started to give it, as well as my interest in the class for my study. Single-variable calculus is a sort of shared experience for Miller students, and if someone has not taken the course, they are surely good friends with someone who has. Plus, it has a large impact on the Miller community due to the large number of students it enrolls. While the aforementioned proof-based calculus class would have been fruitful for my research, I was concerned with getting a healthy sample of various social classes, as the proof-based calculus class is sometimes on the small side and Miller College isn't very socioeconomically diverse. Plus, I had a previous relationship with the professor teaching the section single-variable calculus I observed, which eased logistics and made it easier to create a mutual understanding of my project's place in the classroom.

The single-variable calculus section itself was located in a nondescript academic building that is home to the chemistry department, in a room with chalkboards on two sides. Desks were arranged approximately seven across by five deep, with just a table off to the side for the professor to put their belongings on. It was quite a typical classroom, with tile floors and a few windows in the back. There was a projector and a pull-down projector screen, which I never saw the professor use. There was also a computer off to the side, which I also never saw the professor use. Class was held four times a week, Monday-Tuesday-Wednesday-Friday from 1:00 PM to 1:50 PM. This schedule was the same for all sections of single variable calculus, although at different times of the day.

Participants

Out of twenty-eight students in the class, twenty-five agreed to participate in the study. As it required three rounds of interviews, I suspect that the students who chose not to participate did so because they didn't want to commit their time, but this is just speculation. The full list of those who consented to participate can be seen in Table 4. Since I won't be discussing narrative findings with all of those twenty-five, I will keep them unlabeled in Table 4. Of those twenty-five, I selected ten students that had a good distribution of social class for rounds of interviews. The list of those ten students is in Table 5. They are the focus of my research, but I needed the majority of the class to consent to participate in order to take substantive fieldnotes. As a word of advice, it might be good to keep Table 5 handy when reading my narrative findings and discussion, as I will not explicitly reference a student's social class every time.

Table 4

Student Data

What is your family's social class?	What is your mother's occupation?	What is your father's occupation?	What is your hometown?	What is your class year?	What is your gender identity?	What is your ethnicity? (can select multiple)
Low-Income	N/A	Professor of Biostatistics	Large City, LA	Class of 2024	Female	Black or African American
Low-Income	N/A	Salesman	City, SC	Class of 2025	Female	White
Working-Class	Teacher	N/A	Large City, TX	Class of 2024	Female	Black or African American
Working-Class	Fast Food Worker	Janitor	Large City , NY	Class of 2025	Male	Asian
Working-Class	Department of Defense Auditor	N/A	Suburb, MD	Class of 2024	Female	Black or African American
Middle-Class	College consultant	Financial advisor	Suburb, CT	Class of 2024	Female	White
Middle-Class	Self-employed (micro company)	College Professor	Large City, South America	Class of 2025	Female	White, Hispanic or Latine Origin
Middle-Class	None	Financial Consultant	Large City, IL	Class of 2025	Male	White, Hispanic or Latine Origin
Middle-Class	CFO	None	Large City, MA	Class of 2025	Female	White
Middle-Class	Medical Coder	Engineer	City, GA	Class of 2025	Male	Black or African American
Middle-Class	Dental Hygienist	General Contractor	Suburb, MA	Class of 2025	Female	Asian
Middle-Class	Researcher	Military officer	Large City, S.E. Asia	Class of 2025	Female	Asian
Upper Middle-Class	Real Estate Agent	Contractor	Town, ID	Class of 2025	Female	White

Upper-Class	Small Business Worker	Doctor	Town, NH	Class of 2025	Female	White
Upper-Class	Finance	Attorney	Suburb, MA	Class of 2025	Female	White
Upper-Class	Veterinarian.	Cardiologist	Suburb, MA	Class of 2023	Male	White
Upper-Class	Retired Nurse	Radiologist	Suburb, MN	Class of 2025	Male	White
Upper-Class	At-home mom	Neuro-surgeon	Large City, MA	Class of 2025	Female	Asian
Upper-Class	Unemployed	Real Estate Broker/ Developer	Large City, NY	Class of 2025	Male	White
Upper-Class	House wife	Coder	East Asia	Class of 2025	Female	Asian, White
Upper-Class	Retail Worker	Sales for a startup company	City, UT	Class of 2025	Male	White
Upper-Class	Retired	CEO	Suburb, MA	Class of 2025	Female	White
Upper-Class	Homemaker	Director of Tax and Treasury	Suburb, MA	Class of 2025	Female	White
Upper-Class	Photographer	Retired Lawyer	Suburb, MA	Class of 2025	Female	White
Upper-Class	Pediatrician	Pediatrician	Suburb, NY	Class of 2025	Male	Asian

Table 5

Student Data (Interviewees)

Name	What is your family's social class?	What is your mother's occupation?	What is your father's occupation?	What is your hometown?	What is your class year?	What is your gender identity?	What is your ethnicity? (can select multiple)
Elizabeth	Low-Income	N/A	Salesman	City, SC	Class of 2025	Female	White
Charlotte	Working-Class	Catholic school Teacher	N/A	Large City, TX	Class of 2024	Female	Black or African American
Omar	Working-Class	Fast Food Worker	Janitor	Large City, NY	Class of 2025	Male	Asian
Nazia	Working-Class	Department of Defense Auditor	N/A	Suburb, MD	Class of 2024	Female	Black or African American

Mariam	Middle-Class	N/A	Professor of Biostatistics	Large City, LA	Class of 2024	Female	Black or African American
Markus	Middle-Class	Medical Coder	Engineer	City, GA	Class of 2025	Male	Black or African American
Claire	Upper-Class	Real Estate Agent	Contractor	Town, ID	Class of 2025	Female	White
Luke	Upper-Class	Retail Worker	Sales for a startup company	City, UT	Class of 2025	Male	White
Prisha	Elite	At-home mom	Neurosurgeon	Large City, MA	Class of 2025	Female	Asian
Sean	Elite	Retired Nurse	Radiologist	Suburb, MN	Class of 2025	Male	White

The data in Tables 4 and 5 were collected with a survey in the first month of the semester after they had signed the consent form. Some questions were multiple choice and others were fill-in-the-blank. Students were prompted to put "N/A" if they couldn't answer a question about a parent's occupation. This could mean the parent is not present in the student's life, unemployed, or something else. Full study participants were then selected, attempting to maintain social class diversity, geographic diversity, a representative proportion of gender, and a representative proportion of class year. The larger set of twenty-five students was dominated by upper-class and middle-class students, and in reality, many of those students are elite and upper-class students respectively, as I will discuss in the next paragraph. Moreover, a plurality was from just outside of Boston. As a result of hundreds of years of racism and marginalization, it is sadly no surprise that in selecting proportionally more low-income, working-class, and middle-class students, I also selected proportionally more black students, as well as proportionally less students from the suburbs outside of Boston.

Of note is the self-identification of social class. In the survey, they were given five options: low-income, working class, middle class, upper class, and elite. No student selected elite, although 20% of Miller students are from the top 1%. There were many social class misidentifications. Beyond this, there was a self-identified low-income student whose father is a biostatistics professor, a self-identified middle-class student whose father is a college professor and whose mother runs a “micro company,” and a self-identified middle-class student whose mother is a C.F.O. All are assuredly social class misidentifications. People like to self-identify their social class towards the middle (Bird & Newport, 2021). Plus, at Miller College, it seems that students shy away from being branded as socioeconomically “elite.” Thus, in Table 4, I included student’s self-identified social classes, while in Table 5, I deciphered social classes based upon self-identification and parents’ occupations. Thus, there are mismatches between Table 4 and Table 5 with regards to social class.

Data Collection

I will begin with the secondary component of my data collection—fieldnotes taken during class. I attended approximately ten classes, on a variety of days, although mostly on Friday due to my schedule. These days were spread fairly evenly throughout the semester, although I was not there during the first week, as I needed time to gather consent forms. I regret not being able to observe these days, as the first week of class is crucial in establishing norms. During class, I took general fieldnotes how the professor was teaching, what was being discussed, how things were being framed, what activities were taking place, any professor-student interactions, any student-student interactions, etc. Since only three students did not volunteer for the study, I could keep track of participants relatively easily. Plus, most of my fieldnotes were not focused on specific student interactions. I sat in the same spot every class, in the back in an empty desk among students. I didn’t participate in class in any way aside from occasional eye contact with the professor during their lecture. Also, the professor would sometimes mention my presence. My goal for the fieldnotes was to observe phenomena that students described in their interviews. It would help me understand their experiences more fully, and allow me to ask better questions during interviews. In short, it created opportunities for connections as a supplement to the interviews.

Furthermore, I decided not to observe and take fieldnotes during teaching assistant office hours (TA hours). These are regularly scheduled open work hours, usually in the evening or nighttime, where students can go to a classroom and work on homework assignments with the guidance of a teaching assistant (TA) for the class. This teaching assistant has always taken the class or an equivalent class previously. Often, there are multiple TAs for one session. I have served as a TA for multiple undergraduate mathematics classes. The main reason I decided not to do observations and fieldnotes of TA hours was that the format of TA hours changed drastically at Miller College the year of my research. This was a part of the larger calculus restructuring. Previously, each class would be assigned its own TAs. For fall 2021 and beyond, there are a team of TAs that are collectively assigned to TA hours for all introductory calculus classes. Thus, it would be more difficult to focus TA hour observations and fieldnotes on the participants in this study, as there would be students from all sections, not just the one in this study. Yes, teaching assistants play integral roles in the education of undergraduates, and there is a relative dearth of research about the impact of teaching assistants and professional development in math education (Speer et. al, 2005). I simply had to weigh potential benefits of attending TA hours with the time constraints of research.

The primary piece of data collection was student interviews. One-on-one interviews allowed me to ask students about their subjective experiences, instead of merely observing them during class or looking at grades, the latter of which I didn't even do. Granted, interviews aren't a perfect replacement for an observatory ethnography (Jerolmack & Khan, 2014). Still, they are perfect when resources are limited for research and one is interested in a student's reflections on their past and ideas for their future. Interviews were conducted with the ten students I selected, as described in the participants section. I conducted the interviews mostly in person, with a few Zoom interviews for students who preferred them or weren't on campus during an interview period. I preferred in-person, as it allowed us to sit in silent contemplation less awkwardly and for me to ask follow-up questions more effectively. Nevertheless, there were many enjoyable Zoom interviews. All in-person interviews were conducted in a small conference room in the education department at Miller College. My one formal interview with the professor was done over Zoom.

There were three rounds of interviews, averaging thirty minutes in length. As a quick summary, the first interview focused on students' mathematical pasts, including experiences in their home community and their high school. The second interview focused on students' mathematical presents, namely, the section of single-variable calculus under study. The third interview focused on the students' mathematical futures, both at college and beyond college.

Now, let's talk a little more in depth about my choices for interview questions and sequencing. First, the general flow. I wanted each interview to build on the other conceptually, allowing me to refer to previous interviews and for participants to harken back to their answers consciously or subconsciously. After all, the interviews themselves surely had impacts on the subsequent interviews. The most effective way to do this was to order the interviews chronologically: past, present, and future. This wasn't followed perfectly, but that was the general motivation. Moreover, this sequencing allowed me to not just compare participants' answers, but to potentially compare how participants' answers evolved throughout the time frames that the interviews were focused on. To make this happen, I created questions that stayed relatively thematically consistent across the time frames. With the narrow field of research questions that I was interested in for this study, this happened fairly naturally.

So, what are some examples of questions, and what was I trying to glean? Well, my research questions were laid out in the introduction, and those were my focus. But to answer these, I couldn't dive straight into questions about conceptualizing mathematics' role in participants' lives, how pedagogy might affect this, etc. After all, most participants likely hadn't considered these questions previously, leading to potential confusion about what I was even asking. Even with many scaffolding questions, some still had difficulty understanding what I was asking. Of course, that confusion is data in and of itself. As an example, a scaffolding question I asked was "how can you tell which students/peers are good at mathematics?" This is easy to understand and answer, but also gets participants thinking about their personal relationship with math. These questions led up to larger questions, such as "what is math in the context of your home community?" I asked that same question in the context of the section of single-variable calculus, as well as the context of their future lives. Both scaffolding questions and larger conceptual questions provided valuable data, I did not solely include scaffolding questions to build to larger questions. Pedagogy was often a topic in those scaffolding questions, and I am not uninterested in pedagogy in this study. For those who want more detail, the interview scripts can be found in the appendix at the back.

The single interview with the professor of the class, Professor Murphy, blended questions from all three of the student interviews. I included the "scaffolding" questions as well, because although the professor didn't need them, the answers were still valuable data. For example, in asking about who he considered "good" as mathematics, a typical scaffolding question for the student participants, Professor Murphy gave extended explanations of his ideas around mathematical ability. The single interview with the professor was also a bit longer, both because I included questions from multiple interviews and because they were more comfortable talking about mathematics.

Data Analysis

Identifying themes is the crucial task in qualitative data analysis, and this short section outlines my process for doing so. Every interview was transcribed by a human, and I ended up with a few hundred pages of interviews. These were my data. First, I undertook an inductive review of my data with inspiration from my literature review, reading my data and generating approximately forty "codes" that correspond with recurring ideas, words, themes, etc. With these codes, I completed an initial open code of my data. After this, I narrowed it down to approximately fifteen codes through a process of deleting codes that weren't relevant to my research questions, removing redundant codes, finding relationships between codes, and combining codes under new, broader codes. I then completed a second axial code of my data using these fifteen codes. Finally, I categorized these fifteen codes into my three main narrative finding themes–language shapes knowledge, the value of mathematical knowledge, and the status of mathematical knowledge. All in all, I spent about a month on this whole process.

Narrative Findings

Language Shapes Knowledge

People aren't exactly used to dissecting their own epistemologies. Therefore, interviews with only limited ethnographic fieldnotes might at first seem to provide a challenge in how to understand the epistemologies and knowledge systems of participants. Thankfully, as elucidated in the introduction and literature review, language provides a clear window into epistemology, as language and epistemology create each other. Will will thus both examine explicit expression of epistemological approaches, as I found in Professor Murphy's interview, as well as undertake nuanced inspections of fluency in the language of mathematics present in the class. This framework of fluency will serve dual purposes, building a solid foundation for analysis as well as making the admittedly esoteric concept of epistemology more approachable.

The Professor's Approach

"I do not just do a calculus class and show them how to do problems. I'm trying to make them aware that there's a language part, there's content that's trying to be described," Professor Murphy told me during our interview. In talking about what he was teaching in the course, he mentioned little of differentiation or integration. He said that the formulas one might associate with calculus are "just a little bit of fluff on the side, really. But that's what people think math is." Professor Murphy viewed his lectures as a time to teach the language and intuition of mathematics. He described to me that his students were all early in their mathematics careers relative to the world of math that he lives in, and that although the concepts of calculus may not be that difficult, it was the language that was tripping them up time and time again. Murphy saw his role as teaching them not how to follow a formula so they don't get stuck, but how to ask the right questions when they do get stuck. As he said, “the lecture doesn't help you do it, it gives you an intuition.”

As we will see from student interviews as well, language played a central role in shaping mathematical knowledge in this class. Understanding the language went hand in hand with mathematical intuition and mathematical theory. During classes, he often urged students to put down their pencils and stop taking notes. What he was writing on the board wasn’t a formula to be memorized and applied, but an abstract idea that was meant to build their mathematical intuition, usually in the form of proofs or abstract connections. Professor Murphy would draw shapes, patterns, and write logical sequences to this end, many of which had no direct application to most of the step-by-step homework and exam problems. He had this to say about one class that I observed:

> You saw me apply integration by parts and a couple proofs. And I'm sure a lot of people hated that, because, ‘We are not going to [be tested on that]. Why are you wasting my time?’ No, I’m not. I shouldn't be wasting your time. I'm trying to give you this intuition that this integration by parts gets absorbed in these theoretical understanding later on …

He saw the homework as filling the role of teaching students the step-by-step processes that are needed to find numerical answers to differentiation or integration problems. While that was important, it wasn’t the role of his lectures. He even admitted that you don’t learn much from watching somebody give a lecture, and that math is a participatory sport. Perhaps that is why he asked his students to stop taking notes; he wanted them as intellectually engaged as possible with certain parts of the lectures, even though pedagogical techniques such as in-class group work or turn-and-talks were rare to nonexistent in my observations. This also played out in his student evaluations through the college, where one person even said that the lectures flat-out don’t help with the problem sets. But Professor Murphy took that as a compliment to a certain extent, expressing that the comment regrettably missed the point of the lectures. Instead, they are about developing an abstract mathematical intuition, about “learn[ing] the language of mathematics in calculus,” as he said. Mathematics was being conceptualized and talked about in a new way, with a new language.

An essential nuance to this study was that the calculus program at Miller College was undergoing a renovation of sorts. They hired a calculus coordinator to help oversee the entire program, and had standardized homework and exams for all calculus sections. Thus, the language of the homework, exams, and the lecture were all distinct. There was a communication gap with how mathematics was talked about and done:

> Professor Murphy doesn't make the exams. So, when he's trying to prepare us for the exam, it's... I don't know. It's his class, but it's not his exam. There's some communication gap, I feel like. – Omar

> In the textbook, there's specific ways to do things. I feel like in class, we're taught different ways, but the exams are geared towards testing us on the textbook. So, there's like a little bit of a gap there. – Sean

Lastly, Professor Murphy changed his approach slightly midway through the semester. I did not ask him why he did this, as I was not fully aware of the circumstances until after all the interviews were complete. After all, he is a teacher that is known to be secure in his pedagogy. Charlotte, one of the working-class students in my study that was particularly struggling with the language, “snitched on him” (in her words) to her advisor halfway through the semester, and the way in which he approached and talked about mathematical concepts changed slightly, best illustrated by an increase in references to the textbook, which was rarely mentioned before.

Social Class and Language Fluency

The fluency in the language of mathematical theory in the course could be tracked by social class. Low-income, working-class, and middle-class participants were more often surprised by this shift in language and thus this shift in conceptualization of mathematical knowledge. While some upper-class students also displayed some surprise, they did not voice as much difficulty with the language and math theory. I will start with low-income/working class students, then move along the spectrum to middle-class students, to upper-class students, and finally to elite students.

First, our low-income/working-class students, Elizabeth, Charlotte, Omar, and Nazia. The sole low-income student in the study and the sole low-income student in the entire class was Elizabeth. She described the math in her home community and high school as a straightforward process, and thought that tests were the essential markers for how good someone was at math. It was no surprise then that the most familiar part of the class to her was the textbook, with its "very clear expectations and outlines." It was notable to hear her talk about the textbook in such a way, as it was de-emphasized in the class outside of studying specific question formats for exams. Mariam, a middle-class student who displayed a wide spectrum of attitudes towards the class, and Charlotte both found it difficult to review what was taught in class using the textbook, for as we've already discussed, that was not Professor Murphy's approach. Charlotte even described the textbook as the "Bible for the class," even though the textbook was so de-emphasized that she told her advisor that she didn't like Professor Murphy's methods, spurring him to make alterations and reference the textbook more often and do more practice problems. Thus, that quote revealed the conflict between her own ideas of knowledge and the class itself. Other low-income/working-class students expressed similar relationships with the textbook. Omar talked about how the “grind-y” nature of his high school, and that homework looked like doing a few dozen problems from the textbook and going over them in class the next day, every day. Nazia’s high school took this even further, assigning “a set of 100 questions to do at night.” There was little to no language of mathematical theory.

It’s no surprise, then, that low-income/working-class students were caught off guard by this new language. When asked what the least familiar part of the class was, Elizabeth responded that it was “definitely the theory. Some days we'll just do proofs, and I just haven't really done that before. So, the proofs are new. All the math language used in proofs is kind of new.” She said her previous teachers hadn’t spent much time doing theory, because it wasn’t very important to the subject or the teacher. Omar, one of the students that showed the greatest change in their conceptualization of mathematics, echoed this, explaining that:

> I feel like I needed to use more words explaining things. I feel like I'd never been really forced to explain things, and it's just ... all these symbols and things, they make sense, but you have to spend time with it. It's less grind-y 100% [as compared to my high school math].

Charlotte was also surprised by the short-answer nature of many of the problems, requiring you to explain all of your thought process in words, whereas in high school, there was much more multiple choice, requiring less language of mathematical theory. Nazia’s high school math problems were mostly “all multiple-choice questions and answers or a simple answer, just a number,” although she was undoubtedly the most fluent in the language of mathematical theory of all the low-income/working-class students. Elizabeth also noted that the questions on homework for Professor Murphy's class made you apply concepts in weird situations, and that doing so was an unfamiliar process that she struggled with. The language of the lectures and questions on homework was challenging and unfamiliar on average for low-income/working class students. Granted, there was still an element of memorizing and applying a procedure, especially on the standardized exams, which Elizabeth found "pretty straightforward," Nazia thought were just for testing if "you know how to use a formula," and Charlotte considered similar to what you see in the textbook. "The [exams] were pretty technical, but just the lecture was more theory based," as Elizabeth put it. Hence, for the majority of the learning activity done in the course (lectures and homework) low-income/working-class students were vastly less fluent in the language being used. The language of mathematical theory and intuition created a significantly new way of understanding and knowing mathematics compared to their home community and high school.

Second, our pair of middle-class students, Mariam and Markus. Both displayed a variation in fluency in the language of the class, Mariam especially so. She conveyed less explicit surprise or uncomfort with the language and mathematical theory of the course, for example, but was more surprised by the pedagogical structure of the course. Like Nazia, Mariam was surprised by how often and openly Professor Murphy admitted making mistakes, treating his students more as epistemic equals. This was a "weird dynamic" that she wasn't used to, as they "aren't equals." Mariam enjoyed the class and had a positive experience, but she had to adapt to the unfamiliar structure (I will continue to use the word 'structure' in a broad pedagogical sense) and language of lectures:

> [The structure of class time was] not very familiar. The thing with [Professor Murphy] is that he goes on tangents. So on any given day I'm walking into class, I don't know what

> we're doing. He doesn't give us a chapter to read. He doesn't even tell us what we're doing. I couldn't write the topic of the day, because he touches on so many different things. So that's very unfamiliar, just because I'm used to having more structure when he comes to math.

This is a sentiment that was shared by all but one of the six low-income, working-class, and middle-class students, and only one of the four upper-class and elite students. Notably, that one elite student noted that while unfamiliar, the most unstructured days were some of his favorite days.

Markus found the lecture time unfamiliar as well, but was also more surprised by the language of the class than Mariam. He said that "the way Murphy explains it, he assumes you're supposed to know it kind of," and that "all the work, as formal as you have to do, is kind of frustrating." While all but one elite student found themselves frustrated with the class at times, it is the mention of formality that makes Markus's statement telling.

Third, our pair of upper-class students, Luke and Claire, who were also varsity athletics teammates and friends. It is at this point in our progression along social class that interviewees commonly expressed fluency with the language of the class and how the class was taught, as Nazia was an outlier (although, the middle-class students displayed some significant fluency). Claire made a distinction in the first interview, saying that evaluating prices "isn't really mathematics, it's more statistics." Luke did the same, claiming that the financial world in his hometown is just talking about money, not "actual math." These sentiments were unlike most of the low-income, working-class, and middle-class students, most all of whom considered finances as the essential role of mathematics in their home communities during the first interview, and called it math without hesitation. She was exposed to ideas of distinctly mathematical understanding and intuition in high school, found the lecture style familiar, and found the homework and exams generally familiar.

Nevertheless, Claire still expressed some mild surprise to the language of mathematical theory: “I don't necessarily feel like I had a lot of stuff like that last year. Maybe just scaled down a little bit. I feel like it was more like, do the problems. But it's pretty similar, just less.” Claire talked about Professor Murphy drawing “weird circles” on the board and noted that she could understand why other people with less mathematical experience would want to quit the class. Although she clearly expressed that she “hates words in math” and that she learns better from doing example problems than conceptualizing ideas abstractly (although the two are tied together, of course), she gave a telling story about her experience after a particularly abstract lecture around Euler’s Theorem: “And I came out of class and my friend Luke was just like, ‘Why are you stressed about that? We don't need to know it.’ And I was like, yeah, I guess so.” Such theoretical lectures stressed out most of the low-income and working-class students, who couldn’t find where in the book to learn what was taught in the lecture. Luke was the opposite of stressed. He was arguably the least put off by the language and framing of mathematics in the class, and already conceptualized mathematics as a “different language that makes sense to people … [just] like the English alphabet makes sense to other people.” He felt that for some people, “it just like is a foreign language.” While the class structure was “just a little different,” and the lecture mismatching the homework sometimes confused him, the homeworks had the “same sort of vibe” as in high school, and he felt the class itself framed mathematics in a similar way as his high school classes. He was unfazed, as he was rather fluent.

Fourth and last, our pair of elite students, Sean and Prisha. Prisha gave very similar interviews to Luke, and even expressed that “cooking and measurements isn't a math problem,” just like Claire and Luke. Like Luke, she found the class to be what she was expecting, and a "pretty standard class," which stands in stark contrast to the low-income and working-class students' general astonishment by the language and structure of the lectures. Overall, she had little to say about the language and structure of the course, not because she was jaded and disliked it, but because it wasn't especially remarkable to her. Of course, like any good class, it opened her eyes to things she didn't realize before, as although she had been exposed to "how far everything goes" in math in high school, she felt like Professor Murphy mentioned things like that more often. And although the homework questions were sometimes "not what [she] was expecting," they were "in general … pretty familiar." Like Luke (and to a certain extent, Nazia, Mariam, and Claire) she was unfazed, as she was rather fluent.

Sean's interviews, while sharing through lines with his social class peers, were the most radically different from any student. He was the only student that actively relished in the mathematical theory, even expressing a thirst for more. He had this to say about the familiarity of the structure of class time:

> It's pretty familiar, but it's a little less organized than I like it. But at the same time, the tangents that he does go on are extremely interesting. I find them absolutely fascinating. And those are some of my favorite days.

Moreover, Sean was the only student to express that he would "rather have the exams trend towards what's happening in lecture because it's a lot more interesting," even though the topics in the lecture are chock-full of mathematical theory and the language that goes along with it. He found the calculus course similar to his high school trigonometry course, and was more put off by ticky-tacky grading and step-by-step processes than any abstractness. He found the homework familiar, although he talked about a dual-sided enjoyment and dislike of the very difficult problems. But it was just that nuance that made Sean's interviews so different. I found his way of talking about mathematics to be most similar to the language that I was taught as an undergraduate mathematics major. He was fluent in that language. Only Sean could have given the following response during the last interview, after the class had finished: "... I expected things to be a little bit more abstract, which I got a taste of. But, I assume it'll become less step-by-step and more abstract as I go."

Semester-Long Student Change

Since there were three rounds of interviews spread out over the length of the course, it was very cool to see how some students were shaping and reshaping their conceptions of mathematics, especially since this was the first semester of college for many of them. For those who weren't fluent in the language of the course and found the structure unfamiliar (mostly low-income and working class), there were often marked differences in their responses to similar questions in different interviews. For those who were more fluent (mostly upper-class and elite), the differences were less pronounced and had a distinct flavor.

Accounting for different starting points in fluency (and thus for social class), there was no discernible pattern for semester-long student change. There was the entire spectrum, from becoming jaded towards the class and swearing off the possibility of future, to being enticed by the class and future math classes, with lots of ambivalent attitudes in between. Nazia, likely building upon her foundation of relative fluency in the language of mathematical theory as compared to her working-class peers, was glad that the class made her think of learning mathematics as being "creative with [her] answers" as compared to her high school, where "it was like this is how you do it" with some problem-solving mixed in.

Omar was similarly grateful for how the class changed how he thought about learning mathematics, but he struggled more with that transition. His own words from the third and final interview after the class had finished are powerful:

> [The mathematical learning was] different, less emphasis on answers and more the ideas, the theory. In high school, AP calc, it's just more about get the answer … I think I'm still making that adjustment, I'll be honest … It's hard. It requires a lot of energy, and I don't know. Sometimes I really just feel like I don't have it in me.
>
> I think I learned a lot about calculus. It's the visualizing stuff. It's pretty cool once you get it, but when you don't get it, it sucks.
>
> There was a lot I could have done to make my experience a lot better… I might be the problem here.

While Omar was very open to the new language of mathematical theory, even if he struggled to learn it, others were not. Markus was among the most jaded at the end of the study, stating that he "used to like math, but now, getting to the deep, [he doesn't] like it as much." He even gave a one-word, "No." answer when I asked if the class had affected how he thought about learning math or learning in general during the third interview, despite his conceptualization of mathematics in the first interview clearly being challenged by Professor Murphy by the time of the second interview. Elizabeth and Charlotte didn't show such semester-long change, Mariam said she generally enjoyed being back in a math class, and none of the upper-class or elite students had any significant semester-long change aside from notes about learning habits. Only Sean showed an increase in appetite for college mathematics. There were, on average, less negative semester-long changes in attitudes towards mathematics from the upper-class students, but only slightly so. Plus, changes in attitude were often complex and multi-faceted, not explicitly positive or negative. Some enjoyed the course but realized college math wasn't for them, while others (like Omar) admittedly struggled, but grew in a way that could he (sort of) framed as positive. Thus, social class delineations were less clear.

The Value of Mathematical Knowledge

Going through the rounds of interviews, there were two types of questions that caused significant contradiction within a student's responses. First, as discussed in the previous section, were questions asking participants to conceptualize what mathematics is. That made sense, as it was likely something they had never considered explicitly. The second, related group of questions asked participants to consider the value of mathematics in their home community, in school, and for their own lives. Often interviewees would proffer their opinion without prompt, saying that mathematics didn't matter for them or someone else, but then talked later about how it was used or valued in certain ways. Still, across the instances of contradiction and participants thinking through their opinions in live time, there were notable differences in how students valued mathematical knowledge. This is inherently tied to what they thought mathematical knowledge was, of course, and shall be discussed in the analysis. For now, I will peruse the values of problem-solving, practical applications, making sense of the universe, a way of thinking, and the absence of value, making mention of social class correlation where it exists. The value of math as status will be discussed in the third and final narrative section, as it deserves a more nuanced consideration.

Not Valuable

In this section, perhaps the most frequently expressed sentiment was that mathematics wasn't valuable. Notably, this was expressed most frequently in responses to questions not specifically about the value of math. There were approximately seven significant expressions of math's lack of value in the four upper-class and elite participants, as compared to approximately twenty significant expressions in the six low-income, working-class, and middle-class participants. The participant that expressed math's lack of value the most often was Elizabeth, the sole low-income participant. In talking about her high school, she described an interesting dichotomy between the teachers and the students:

Elizabeth: Well, I think there's... Our math teachers obviously cared about it more and thought it was valuable just because you are learning problem-solving skills, and that's important, I guess. So they would definitely see it as more important for not just school, but just having a good brain, being able to work through things. I guess my science teachers would probably agree with that and just thinking logically.

Silas: Was that sentiment shared by the students, or was that…?

Elizabeth: I don't think so. The majority felt like, at a certain point, math was no longer useful to your day-to-day activities. So, I mean, I guess it was helping us learn problem-solving skills, but it just felt, I don't know, irrelevant.

This exchange came during the first interview when talking about hometowns and high schools. No upper-class or elite students explicitly said that any pre-calculus math was not valuable at all as Elizabeth did. Yet, working-class Nazia did, saying that "no one really talks about it outside of school-ish and yeah, no one really talks about that. It doesn't really play a big role in anything, I don't think." Working-class Charlotte responded similarly to questions about the value of math in high school and home, saying that math was valuable "never. Just never. The only thing that ever had value was learning how to do addition, subtraction, division, and multiplication." As a middle-class participant, Mariam had the highest social class of those that expressed these views about home, saying she doesn't "think [math] is really defined outside of the classroom nor is it really valued outside of the classroom." She thought that of both adults and peers.

People of all social classes, however, mentioned the lack of value of the math they were learning in Professor Murphy's class at a similar frequency and significance. Many of them said the class gave them confidence, helped them develop good learning habits, or gave them a skill such as coding Overleaf, but beyond that, there was little to no value for the rest of their life outside of non-mathematical classes. Of course, with all of these responses about math's value or lack of value, participants frequently contradicted themselves. It was common for a participant to talk about the value of the class for them, then claim it had no value when asked directly. Thus, it was difficult to track and pin down participants' opinions.

Practical Applications

The next most commonly expressed value of mathematics was, by far, that of practical real-world application. This was shared by every single participant, although it came in many different flavors. Some participants gave more financially focused practical applications, such as Markus's responses to my questions on math's value in his home community:

> I'd say [math's value is] when they're doing some business, like either dealing with money or cars or anything like that, buying stuff, or saving or investing.

> How to deal with money and be able to survive, because I feel like money is math, basically.

> I guess you could say [math] goes you a long way, dealing with personal stuff, personal finance. Able to be living off of a personal income you have, I guess you could say.

This was shared by Prisha, saying that “a big thing is money, obviously, because that's something that affects everyone's day-to-day lives,” as well as Elizabeth on the other end of the social class spectrum, saying that she thinks math’s value is “with financial decisions, probably. It's not really applicable, from what I've experienced outside of that. Just figuring out bills and, yeah figuring out budgets.” Many more expressed this financial practical application, including Omar and Nazia significantly so, Luke and Claire with the caveat that financial math is more statistics than math, and finally Sean, Mariam, and Charlotte not mentioning it at all. Perhaps something was happening here with social class, with more low-income, working-class, and middle-class participants mentioning daily finances more often as a core value of mathematics, but the correlation was moderate at best.

Finances, however, were only one type of practical application mentioned. The other broad type was described by Professor Murphy as what he thinks students conceptualize math as:

> Maybe you can attach meaning to these formulas, physical meaning, and use these in applied ways that make the world go around, which it does. And that's all true, but that's not what math is. That's just a little bit of fluff on the side, really. But that's what people think math is.

The attitude of math as useful formulas for other fields was found in every participant. Students valued math as a means for completing the tasks of other subjects and college majors, especially for economics and the sciences. Plus, many of them thought about the careers they might have in the future or the careers the adults in their home community currently have, and the math that is needed for those jobs. When talking specifically about Professor Murphy's class, I could label these as applications of calculus, even if that undersells the benefits of a calculus course. But for most low-income and working-class participants, this was the extent of math's claimed value. The following two sections represent values that were less commonly expressed.

Problem-Solving

This section focuses on problem-solving, and the next section focuses on mathematics as a way of conceptualizing the world. Of these two, problem-solving was the more commonly expressed value. Professor Murphy described the process as:

> Very frustrating. It's like doing a hard puzzle. And the more deep the problem is, like research level, it becomes a super hard puzzle with many layers. But if you're doing an exercise from a book, it's an easy-to-medium difficulty puzzle with very few layers. And most of the layers are spelled out in the preceding chapter. In a calculus text, most people will navigate that by going from the problem to reading the examples, and then they're digesting some of the layers that they need to solve that particular problem in doing that.

The prophet of problem-solving was one of our two elite participants, Sean, and only he and Mariam mentioned it in the first interview about their high school and home community. Sean kept coming back to it countless times in the interviews as a gauge for who is good at math, the value of math, and the core of what mathematics was to him. For example, he mentioned that he thought his chemistry-major roommate would be good at math because they were good at working through the logic of problems on a page. Sean said he "actually enjoys the questions and problem-solving," and was interested in taking math classes because he "enjoy[s] critical thinking and problem-solving." He described his journey:

> Well, I initially took it because I felt like it was a box I needed to check, but now I'm interested in taking linear [algebra] because I just enjoy the process of solving problems. So I think it just improves your overall thinking skills, and I think that's the value of it.

The other elite participant, Prisha, was not nearly as enthusiastic about problem-solving in mathematics, only mentioning it as a "way of thinking and figuring out problems using numbers." Claire was similarly ambivalent, only saying that calculus might help her relate to or think of things differently in a statistics class. Luke, the other upper-class participant with Claire, was more enthused, appreciating Professor Murphy's explanation of math as a mind game:

> [Professor Murphy] said today in class that everyone should continue taking math because it's a way to really force your brain to learn something new, and conceptualize something in a different way… I think it's a pretty cool way to think about it. It's definitely a different way to work your brain and a different side of your brain than the literature or something like that. So it's kind of interesting, it's like a puzzle, he said.

Moving onto the middle-class participants, Mariam mentioned briefly that math is "just a way to solve problems, hypothetical problems … it just shows your ability to solve problems that might or might not include numbers." Markus did not mention problem-solving at all. For the low-income/working-class participants, Charlotte did not mention problem-solving at all, and Elizabeth mentioned it skeptically in the first interview, as already discussed. Elizabeth also noted that the class had an emphasis on both problem-solving and memorizing procedures, although ultimately concluded that memorizing procedures was the more apt way to describe the class and what value it had to her. Omar did not mention problem-solving at all during the first interview, although he also appreciated Professor Murphy's framing of math as "really hard mind games." Omar had one of the most significant changes in perspective towards mathematics throughout the semester, as has been discussed. Lastly, Nazia was the strongest proponent of problem-solving of the low-income/working-class participants, which isn't surprising considering her relative fluency in the language of abstract mathematics for her social class group. In the third interview, looking back on the semester, Nazia saw problem-solving as one of the main takeaways. She asserted that her high school taught math in a similar problem-solving manner, although I would put an asterisk next to that assertion based on her description of the day-to-day of her high school math experience. On the whole, although problem-solving didn't have a very significant correlation with social class, it is notable that the participant that emphasized it the most, Sean, was elite. Furthermore, when combined with the next value of mathematics, we start to see a fuller picture.

Conceptualizing The World

This final value of mathematics came from more generalized expressions of mathematics being an integral part of the world around us, and that learning math might help us understand the world more fully. This wasn't to be done through formulas, but through a sort of mathematical creativity. After one of the classes I attended, I was talking with Professor Murphy about how he conceptualized his teaching and doing of mathematics. He told me he thought of himself as an artist, and that he was trying to impart some of that framing to his students. He wanted them to look at the world in a way that was logical but creative, that draws from the insight that math can give. This was by far the most rarely expressed value of mathematics throughout the thirty participant interviews. Sometimes, it was interlaced with other values, such as problem-solving in Sean's assertion that math helps "how you process things, and looking at things from different angles." Other times, it stood alone.

While the social class correlation with problem-solving and practical application were both somewhat muddied, the social class correlation with the value of conceptualizing the world was much clearer. Of the four low-income/working-class participants, only Nazia expressed this value of mathematics at all, saying that there were real-life situations that math helped her understand a bit better, and that it was "not necessarily the number, but just like the reason behind something." Yet, she also said that in her high school and hometown "math is very specific," and harder to gauge aptitude unless literally solving an equation. Meanwhile, her fellow working-class participant, Charlotte, once showed active disbelief that math changes how people might understand the world, as she said she looks at people who she considers good at math and she thinks: "Do you see color? I don't know." The middle-class participants were once again split, with Markus not valuing math as a way of conceptualizing the world, and Mariam doing so considerably, if tentatively:

> People say it's not really useful in life, but I think it will be. I just don't know how … It's unexpected sometimes … I'm very sure that it's not going to be like, 'Oh wow. We need to take the derivative of this. I learned this in Calculus.' It's not going to be that thing. But more of the stuff that we might have learned conceptually and like… Yeah.

This came from the third interview, although she even expressed similar ideas in the second interview, saying that calculus is "not really something that you would think about, but it's just a different way to conceptualize real-world problems that other people might be solving." A fairly strong statement.

Next, our upper-class friends, Luke and Claire. Of the two, Luke expressed this value less explicitly. Similar to Sean, he blended this value of mathematics with that of problem-solving, as the two are intimately related. Luke’s quotes in the previous section on problem-solving can be easily reframed as a conceptualization of the world, and he additionally claimed that math was something in the back of some people’s minds that allowed them to make sense of things. Still, this was weaker than Claire’s expression of the value. She said this talking about how math might help a potter see their work differently:

> You still look at your bowl, and it's hopefully symmetrical, right? That's something like mathematical, right? … Like I feel like [math] still ties in somewhere, because you're making 3-D objects, like the cup can't have a hole in it, and somehow your handle needs to hold the mass of the cup. You're using math a little bit. Like they probably don't think about it, but if they did think about it, they'd probably be like, ‘Math did teach me something.’

Although she did so with a hint of irony, Claire noted that “the physical world is made up of math, or something like that.” For a student in a course where almost everyone is just trying to pass to check off the requirement, such a statement was rare. Claire reiterated math is “just a big part of the world” in a different interview, and that “it kind of applies pretty broadly, when [you] think about it. It’s weird.” Weird indeed.

Finally, Sean and Prisha with our elite social class. Similar to the difference between Luke and Claire, Sean expressed the value of math as a way of conceptualizing the world less than Prisha, and as already mentioned, he interlaced it into his extensive discussions about problem-solving. More than anything, to Sean, math was a way of improving “overall thinking skills.” While this was mostly framed around working through specific problems, such skills are what gave his dad the ability to be “very, very good with big picture stuff.” Sean saw problem-solving not as a way to get a test question correct, but as a way to understand the world around him. If you recall, Prisha did not talk much about problem-solving skills. Instead, she was, along with Claire, the most extensive in her expression of math's value in conceptualizing the world. Starting in the first interview at the beginning of the semester, she saw math as a way to perceive that there are “certain things that … don’t make sense in the scheme of the world.” When asked directly about math’s value, she waffled, saying that math was an inherent thing that just is the way it is. To her, math is used to “evaluate and not categorize, but make sense of the world.” She saw some “fundamental parts of math” that are “never going to change,” that “play into our lives,” and that are “involved in everything.” And to Prisha, that involvement wasn’t limited to practical applications:

> [Math] affects everything in some sort of way. I don't know what I'm thinking, but it's like physics is always happening, and that's math. That's the best thing I can think of, it's less explicit, more understood in a way, where it's like, we're not going to do out a problem, but we know things are happening because of math.

She truly saw math as a lens for everyone to see the world in a new way, even if we don’t think about it all the time.

The Status of Mathematical Knowledge

Someone experienced with student attitudes towards mathematics might be surprised that there was little to mention of the value of math as a means to an end, for reasons outlined in the discussion section. Well, fear not, discerning reader. Such attitudes were so prevalent that they deserve an entire (if slightly smaller) narrative section. They included discussions of grading, math's social status, and competition, as well as the rare foil that math is for everyone. The first three of those values flowed in and out of each other, supporting and complementing each other. For example, there was competition over both grades and status, and grades bolstered the status that math afforded. Thus, it is especially arbitrary to divide up the status of mathematical knowledge into discrete sections. Most of the quotes in this section were coded identically, as "means to an end." That being said, categorization is still useful for digestion.

Grades

Arguably, the backbone of students seeing mathematics as a means to an end was grading. Although not mentioned at all by Professor Murphy as a value of mathematical knowledge, grades and standardized test scores were the participants' typical answer to my plethora of questions about math's value to them and others in different contexts. These included both GPA and SAT/ACT scores. Both were seen as means to get into good colleges and secure good jobs, and the value of mathematics was merely to score well on mathematics exams. Usually, when asked about math's value or who they considered good at math, students gave short replies about "getting good grades," assuming that I understood the subtext of GPA, transcripts, getting into good colleges, getting good jobs, etc. Sometimes, I would draw out these details from students, but eventually it became monotonous and pointless. Such short replies included "it's always everything about the grade" from Mariam, the value of math is "really just for my GPA" from Elizabeth, and people "wanted to get a good grade so you get into a good school kind of thing, that was like the end all, be all" from Luke. Participants of all social classes expressed these views, including the elite of Prisha, who expressed that math was valuable for:

> Grades lead to getting into college. And I went to, like I said, I went to a private school that was very academically rigorous and there was an expectation for people to get into

> good colleges, and it's 100% college rate, and seven kids in my grade out of 120 went to Harvard. I don't know. It's just that's the atmosphere.

This was shared by working-class participants, such as Omar:

> Honestly, it feels like a means to an end. It's like you don't really do it for enjoyment or curiosity, just if you're good enough at this your options increase, your opportunities increase, you get to choose whatever college you want to go to. That in return just increases your earning potential, things like that. You can really provide for your family. Yeah, it's more like a means to an end.

Although they had different motivations underlying their valuations of mathematics, they both saw math as a means to an end. While those different valuations may seem sociologically significant with regard to social class, I did not have the data to back up such differences over the entirety of the interviews. Nevertheless, in the end, there were seven pages of quotes treating math as a means to an end from low-income/working-class participants, and under five such pages from upper-class/elite participants. With all these quotes, I could go on, but I won't. I'd just be repeating myself.

Secondly, related to grades, was the value of math classes as a distribution requirement. By this I mean to say that participants often expressed that the only reason they took this class or other math classes was to satisfy a major requirement or requirement for graduation from high school or college, and to acquire the status that went along with that degree. In the final interview, every student except Sean gave this answer to my question of why they took this class, although Claire hemmed and hawed for a moment. Note, both are elite/upper-class, although Claire still mentioned grades and requirements often. Sean is the only participant who did not.

It was rare for students to rebel against the framing of math as a means to an end, it did occur. After all, Professor Murphy said he tried to tell his students “that good learning and education is a growing experience. Not an experience where you're supposed to demonstrate that you get it.” The weakest type of rebuke of math as a means to an end was an expression of interest in how the math was done, not just the answer. But often, such interest was only due to tests that required students to know how to do the math. Prisha showed some hesitancy in equating getting good grades with the value of mathematics, but only during the last interview. Omar showed a remarkable transformation, as although he still admitted he took the class to fulfill potential major and/or graduation requirements, he started to feel that “not everything is about getting credit for it,” something he said he didn’t feel at all in his home community or high school:

> Back home, it didn't feel pure. I don't know. You do it to look impressive on apps, go to engineering school or something, you know what I mean? It's like means to an end. But I feel like just by the merit of you being at Miller, I feel like you've already made it, and just expand your mind for the sake of expanding your mind. And you could do that through math very well.

Sean was the only participant to actively complain that the math class was too grade-focused. He complained that if he missed a negative sign on an answer for homework, he got eight percent off, whereas in high school, if he showed he understood the process and gave full effort, he would get full marks. To him, homework felt like quizzes, more focused on the answer than on the work. He preferred his high school, where “they would more like assessments that do not impact your grade, they give you better ways to keep track of where you are.”

Competition

Implicit in conversations around grading and "good" colleges, but also expressed explicitly on occasion, competition was an integral link between grades and status. I will not undertake an examination of the implicit mentions of competition, as that discussion is better suited for analysis sections. Here, I will focus on the explicit. Interestingly, such mentions were confined to four participants—three working-class and one middle-class. Of those, three were in reference to the culture of their high schools, while Omar's was in reference to "gauging where you are in the competitive market" of schools and jobs. Omar's statement has more in common with the sections on grades and status, so I will focus on the high school math culture of the other three participants—Charlotte, Nazia, and Mariam.

While each of these three cited different flavors of academic competition in their high school, they were the only three to mention it explicitly. But it was striking when they did so. The most toxic competition was in Charlotte's high school:

> People were a little selfish in high school. There were people who would ask me for help, and I would help them, like literally explain the stuff to them. But then, when we asked them for help, they'll give you the partial answer because they don't want you to know everything. Because, they don't want you to get a higher grade on the test than they did.

This competition was a part of the culture of the IB program at her school, which she described as "welcome to torture," but it was justified because it prepared students for the rigors of college. This was similar to Mariam, who said she felt "like it was like a competition. Sometimes it got heated. Everyone just wanted to show everyone else that they were smart … but it was mostly inside the classroom." These quotes from Charlotte and Mariam showcase a student versus student competition inside the classroom, not to the benefit of all, but to gain personal benefit or show their own status.

Nazia's case was somewhat unique, as I will talk about more in analysis. She went to a high school within a community college that gave students a pathway to graduates with an associate's degree. There were no extracurriculars or school dances or anything like that, but lots of focus on competition and testing. Similar to Charlotte and Mariam, Nazia described a culture where "everyone was like, I'm good at math, I'm better. I'm better and better." However, the competition was also different from Charlotte and Mariam, as it focused on the status of the school itself as well. They were the "number one school in Maryland," and were "trying to stay there." This was done both through exams and a national high school math competition that the school would win every year. Even the pedagogy had distinctly competitive tinges, as they had multiple choice math relays where you competed against your classmates, and their teacher told them they had to do 100 math problems as fast as possible to do well on the ACT. Her description of her high school's math pedagogy conflicted with her claims that problem-solving was emphasized.

Status Symbol

The first two subthemes of grading and competition were somewhat expected, or at least, I was not surprised to find them in the data. Math as an explicit status symbol, either externalized or internalized, was something I first started to recognize as a common sentiment in the final interviews when asking how this class will be a part of their lives after college. Across a variety of social classes was the idea that "math is recognized as being kind of like a hard area of study," and thus "looks good on a transcript," as Claire put it. Even Professor Murphy talked about the perceived special status that mathematics has among many employers and academics:

> I always give a blurb [to the students] about how when you apply for jobs or when you apply to graduate school, the people who are evaluating your resume are going to be looking to see what kind of courses you take in order to judge whether you've challenged yourself or not. And they'll have some biases. And I said mathematics generally is a big eyeopener, for not only the people who studied math. Everybody knows that it's a hard subject. I think it's just generally accepted that it's a difficult, hard subject. And so it's held in high regard by those who are evaluating you. And so, on one hand, it's great for your resume. So if you enjoy it, you should continue.

Students understood this. They saw it as Luke did, as “that one hard math class that I took freshman year.” There is a status associated not only with a college degree that shows you fulfilled certain distribution requirements, as Omar noted, but also with taking a college calculus course in particular. Charlotte and Mariam, both of whom emphasized competition, also emphasized that they would be able to brag about having taken and passed this class, as for some reason, math is held in a higher regard. After all, we already heard Charlotte talk about she considered math majors as “gods or something.”

Talking about mathematics like this went beyond non-majors taking an introductory course. Some participants even internalized this status symbol, expressing that it was about proving to themselves that they could take and pass such a course. Omar felt proud of himself for holding his own in the course, Claire didn’t want to be the person who never took calculus so she could feel better about herself, and both Charlotte and Mariam showed traces of this internalization. But there was a fine line between Professor Murphy’ observation that society seems to hold math in a higher regard, that it looks good on resumes, and some participants’ expressions that math was a marker of intelligence. Sometimes participants' answers tended towards the former, sometimes the latter. But not once did anyone question why math was held in such a regard. It was accepted as part of the "system that's been implemented," as Charlotte said.

Math Is For Everyone

Lastly, the foil. If there was an antithesis to mathematical knowledge as status, it was the theme that math is for everyone. Of course, one could still see math as having a special status and not believe in being inherently good or not good at math, as that was exactly the point of view that Professor Murphy took. Thus, perhaps instead of considering this section as the foil, perhaps we should consider it as a nuance. Yes, math still may have a special status, but it is not a marker of intelligence or reserved for a select few. In the first interview, Charlotte thought that "if you're willing to learn [math], then you could be good at it." But she was also talking about the cutthroat competitive nature of her high school math, and by the last interview, she was talking about math majors as "gods." Claire made a similar remark to Charlotte in the first interview, saying that her friend back home isn't "bad [at math], it's just math is just something she struggles with." These were, however, both rather isolated comments. Interestingly, Markus thought that there were some kids in his public school that were bad at math, but there was nobody in his private school that was bad at math. Nevertheless, I believe he was basing this off of grades as opposed to some grander sentiment of a democratic mathematics.

The only other students who had trouble answering questions about whom they considered good at math were Prisha and Sean, although Prisha considerably less so. She mentioned that asking a lot of questions may be a marker of being bad at math, but it also may just mean it takes more time for them to understand. Besides what classes a person is taking and what they are interested in, she doesn't know if she'd be able to tell who is good at math. Yet, earlier in the same interview, she said tests helped determine who is good at math. As I said earlier, participants often contradicted themselves as they thought through their opinions. That brings us to Sean, who was once again the outlier. He was most similar to Professor Murphy, expressing in both the first and second interviews (the interviews in which I asked how participants determined who is good at math) that math is for everyone. Sean didn’t consider people specifically bad at math, instead making a delineation between qualitative and quantitative people. When asked about his high school peers, he wanted “to say that all of them were good [at math]. It's just that some of them underestimated themselves a little bit.” Sean even asserted that “you don’t have to be in a super mathy job to use math,” framing math as something that is useful and accessible to everyone. I don’t want to overstate his position, as he would also make the offhand comment about not wanting to fail, but his interviews were generally stunning as compared to the rest of the participants, especially those who were low-income/working-class.

Discussion

In surveying the landscape of theory around education and epistemology, as I did in the literature review, we find a distinction between works that use knowledge as a framework and works that use language as a framework. This pair of concepts should be integrated into one if the goal is constructing a holistic picture of reality, but in doing so one might lose some power and nuance found in the individual theoretical frameworks. So, I will be using them separately in this analysis. I look at specifics not to ignore the larger picture or the interdependent nature of all things, but to make sharper critiques and suggestions. With this impetus, our analysis will be divided into two sections, about language and about knowledge, corresponding quite neatly to the first two sections of the discussion, respectively. The third section of the discussion, about the status of mathematical knowledge, will be minimally integrated into both sections of analysis and significantly integrated into the conclusion section to follow.

First, however, I need to talk about the participants. A minority of the low-income/working-class participants at Miller College came from a neighborhood public school, instead going to other high schools. Indeed, none of the low-income, working-class, or middle-class participants went to the normal track in their local public school:

Table 6

Name	Social Class	High School
Elizabeth	Low-Income	Magnet school
Charlotte	Working-Class	International baccalaureate high school
Omar	Working-Class	Specialized high school
Nazia	Working-Class	Magnet/specialized school
Mariam	Middle-Class	Charter school
Markus	Middle-Class	Private high school (for last two years)

To different degrees, every one of these participants was a member of Anthony Jack's (2019) "privileged poor," as their high schools bestowed some of the social capital, cultural capital, and habitus of the upper-middle-class, upper-class, and elite. Communication in all shapes and flavors is an essential component of that capital, as it allows not only for students to let their needs be known and to access unspoken resources, but to participate fully in knowledge systems that revolve around elite language and conceptualizations. In particular, Nazia went to a high school situated within a community college with the ability to take college courses and graduate with an associate's degree, and Omar went to one of the top-ranked high schools in New York State. Their status as privileged poor helps us understand their unique reactions to the calculus course as compared to a participant such as Elizabeth, who went to a run-of-the-mill, albeit well-off, magnet school. Nazia was the most comfortable with the language of the class of all low-income/working-class participants, and Omar learned to appreciate the language more rapidly than Elizabeth, Charlotte, or Markus.

Granted, every person is an individual with quirks and differences that can't be explained by systemic analysis. I need only look at myself, who came from a middle-class family but has learned to revel in the language of elite theoretical mathematics, or at Mariam, who is the daughter of a biostatistics professor but self-identified multiple times as low-income. Sadly, this study does not have the time or resources for an analysis of individual agencies in response to systems, apart from cursory mentions.

Language

Learning the mathematical proof as a dynamic doorway to the world of mathematics is a question of teaching epistemic fluency in the language of pure mathematics, allowing the student to create and recreate mathematical knowledge (Solomon, 2006). As argued in the literature review and as seen in narrative findings, such language is classed. I believe such differences in mathematical language can be understood through the lens of both Basil Bernstein's (1964) and Annette Lareau's (2011) work. At first, I was pessimistic in using Bernstein as a theoretical framework, as the question I asked about the amount of work participants felt they had to show for classwork produced no social class correlation. The problem was, in mathematics, showing your work has at least two different meanings. First is the classic "show your work" of middle school, where students show every step of algebra line-by-line. This is more or less drivel, meant to make sure students have adequately memorized the prescribed steps of solving a particular type of problem. The second meaning is the showing of your thought process, explaining to the reader what you are doing and why you are doing it in words. This second meaning is the process of proof-based mathematics, as is more akin to Bernstein's (1964) elaborated code that is explicit, thorough, and doesn't ask the audience to read between the lines to understand. As a result of these dual meanings, my question about showing one's work wasn't nuanced enough. Thankfully, participants talked at length about the type of work they did in response to other questions.

Essentially, I am arguing for an interpretation of Bernstein's (1964) elaborated and restricted codes such that mathematical language can be seen as elaborated or restricted. Due to the multiple meanings of "showing your work" in mathematics, I cannot merely rely on the amount of work shown. Instead, I have to focus on the type of work shown, as well as who struggled to show certain kinds of work and do certain kinds of problems. As already mentioned, I am equating elaborated code to the type of abstract, theoretical, formal, proof-flavored mathematics that requires students to explicitly detail their mathematical thought processes. I will call this *elaborated mathematical code.* Meanwhile, I am equating restricted code to the plug-and-chug, show your algebra, traditional textbook problem mathematics that assumes an understanding of processes and only asks for the application of the formula. I will call this *restricted mathematical code.* This is a dichotomy that is often oversimplified in mathematics education, yet it still holds immense value, as in Anyon's work around social class and knowledge. Moreover, by naming it using Bernstein's codes of knowledge, I can talk about the dichotomy not merely as a pedagogical one, but as a cultural one with sociological ramifications.

Granted, the math done for Professor Murphy's class was not purely elaborated or restricted. When talking about his class, he positioned his lectures as intuition-building activities that were meant to give students an understanding of the logic and language around a concept so that they could solve problems using that intuition instead of relying upon a memorized formula. Moreover, some of the homework required students to explicitly detail thought processes, not just show the algebra. Both of these can be categorized as elaborated mathematical code. The exams, however, were better categorized as restricted mathematical code. As multiple participants emphasized, the exams felt thematically disconnected from the rest of the class, as they were heavily centered around the application of formulas from the textbook as opposed to requiring students to explicitly detail their mathematical thought process. They still had to show work, yes, but it was a different kind of work. Students studied for the tests by reading textbook chapters, memorizing formulas, and memorizing step-by-step problem-solving processes, forgoing any elaborated mathematical code. The Friday quizzes that Professor Murphy gave were similarly restricted in their code. Remember, the standardization of exams across the many calculus sections was something relatively new at Miller College and reflects Apple's (1982) analysis that teachers are being deskilled and proletarianized as standardized curriculum takes more and more control of their classrooms. Nevertheless, while much of the mathematics of Professor Murphy's class laid on the spectrum between these two extremes, as does much mathematics, we still saw social class patterns of fluency with elaborated and restricted codes that Bernstein theorized, as the codes of mathematical language used were different from the ones many students learned in their prior schooling.

In short, low-income/working-class participants felt like the elaborated mathematical code of the class was alien. Omar's "grind-y" math classes in high school are a prototypical example of a restricted mathematical code, as are the multiple-choice style problems of Nazia and Charlotte's high schools. While Elizabeth thought that math was a rather straightforward subject at the beginning of the semester, the elaborated mathematical code of the class caused self-doubt and an increased antipathy towards math. She struggled to "apply concepts in weird situations," and Omar realized he had never been forced to explain his math in words. They had both only been exposed to restricted mathematical code. I am sure, of course, that all students had to complete multiple-choice problems and plug-and-chug processes in high school. As Bernstein (1964) points out, restricted code is used by all social classes, restricted mathematical code follows this as well. All students are able to compute rote plug-and-chug processes, and those processes were present in the class.

Nevertheless, the upper-class and elite students' fluency in the language of mathematical theory of that class is a result of their exposure to *both* elaborated and restricted mathematical codes, whereas children of lower social class only use restricted codes. As members of higher social classes, Prisha, Claire, and Luke showed the ability to differentiate between the two, labeling activities such as finances, prices, and cooking measurements as not really math problems. In Bernstein's (1964) language, they meant such activities didn't use elaborated mathematical code and thus weren't true mathematics. This ability to draw a clear line between ticky-tacky practical applications of algebra and larger problem-solving processes was primarily found in the upper social classes. Thus, they expected the mathematical language (elaborated mathematical code) of Professor Murphy's class. Because most low-income/working-class and some middle-class students didn't have as much exposure to the elaborated mathematical code of higher social classes, they were more flustered by the language of the class. This explains why some of these same participants also had negative reactions to the class as a whole, as they likely felt confused and trapped by their new inability to speak the language of mathematics (Aires & Seider, 2005), even if the social class delineations for semester-long student change in attitudes toward math were not crystal clear. Still, in one of the most striking moments of the study, Sean, an elite student, complained that there wasn't enough abstract theory, that there wasn't enough elaborated mathematical code. He hoped to take more math classes where there would be more of it, reflecting that in his mind, the class I observed was still just an introductory calculus class. For those students of lower social class, it was not just an introductory calculus class, it was an entirely new field that required unfamiliar habitus (Bourdieu 1987). Admittedly, as a whole, the scope of this study doesn't provide sufficient data to back this theoretical framework beyond a doubt. Yet I discussed it extensively because it is one of the most illuminating and potentially powerful.

There are nuances yet to be figured out, of course, as I feel more comfortable making normative judgments about the limited value of restricted mathematical code than the value of regular restricted code. Restricted mathematical code is different from general restricted code because the shared understanding needed for true restricted code use does not exist in many situations where restricted mathematical code is heavily emphasized. In other words, the students apply the formulas without explanation just as restricted code would suggest, but they do not understand why they used that formula. It is a type of *false* restricted mathematical code. In Bernstein's restricted code, the subject has that understanding, even if they cannot express it in the way the elaborated code of elite spaces would require.

Nailing "false" onto restricted mathematical code as a prefix may seem to be a put down of the language systems of the low-income and working classes. To this, I will say two things. First, false restricted mathematical code is found throughout all social classes, as with all restricted code. Second, as an educator, I would argue that mathematics education in working-class schools is particularly repressive, thus warranting a stronger critique. I have thus far avoided such pedagogical critiques, as academic research so often frames the education of low-income/working-class communities as inferior, deprived, and deficient. While there is truth in this as it relates to inequitable funding, we cannot critique teachers in these communities for trying to provide students with the traditional skills they rightfully see as the way their students will be able to access the dominant culture. False restricted mathematical code can still get a student into college through SAT scores, and it is perhaps the most efficient way to do so. I am inspired by Delpit's extended critique of white progressive educators in her book *Other People's Children* (2006), for education is so complex because it is stuck between what the world is and what we want the world to be. In theorizing about the best educational practices, we must admit that what is best over the long-term might not be what is best for our students here and now. There are pedagogies that seem repressive to some, but are a tool to fight oppression for others, and false restricted mathematical code is one of these pedagogies. I will continue this discussion in the conclusion.

Lastly, as a straightforward addendum to Bernstein, is Annette Lareau's (2011) theory of distinct parenting styles for low-income/working-class families versus middle-class, upper-class, and elite families. The "natural development" parenting style of the former emphasizes directives, listening to authority, and unstructured leisure time. The "concerted cultivation" style of the latter emphasizes negotiation, questioning authority, and leisure time structures by organized activities. We can clearly see the effects of these two parenting styles' approaches to authority in the data. Low-income and working-class participants were more eager to treat the textbook as the arbiter of mathematical knowledge, and were more put off that Professor Murphy allowed students to correct him so frequently and openly during lectures. When Charlotte complained about his teaching style midway through the semester, it was largely because he wasn't connecting his lectures to the textbook consistently. There was by and large a greater reverence for traditional mathematical authority among participants of lower social class, even if all students felt relatively comfortable approaching Professor Murphy inside and outside of class. Lareau made further observations of independence and entitlement among children from different social classes due to these parenting styles, but I found no data to reflect those themes.

Knowledge

There were times, however, when the interview and the participants used the language of knowledge as opposed to the language of language. As was seen in both the literature review and in the discussion, mathematical proofs require values and norms such as decontextualized reasoning, a more elaborated code, and the manipulation of a priori truth. Mathematical knowledge is framed by sociomathematical norms (Yackel & Cobb, 1996), and those norms were more or less familiar to those of varying social classes in Professor Murphy's class. This resulted in classed conceptualizations of mathematical knowledge, just as Anyon (1981) found in her seminal studies decades ago. While I seemingly asked questions about the *value* of mathematical knowledge, I was really finding out what mathematical knowledge meant to them. After all, the meaning of knowledge is inherently tied to its stated value. Plus, it's not like I could ask students what mathematical knowledge meant to them and expect responses beyond vague statements about numbers. Asking about *value* was my way around that problem. Hence, while the discussion section was focused on value, the analysis will be focused on extrapolated student conceptualizations of mathematical knowledge.

Why were students of lower social class much more likely to express that math wasn't valuable to them? Only considering the number of mentions, this was one of the most significant findings of the study regarding social class. I argue that this was akin to the theme of *resistance* that Anyon found in working-class schools. Students explicitly and implicitly fought back against the curriculum in a variety of ways in her study, as did students in this class. Many of the comments about math's lack of value came off-the-cuff, unprompted, and not in direct relation to Professor Murphy's class. Students of lower social class more often resisted mathematics as something valuable, as they likely implicitly recognized the system of oppression as work. They felt it, as one feels a dread towards mathematics. Granted, mathematics is one of the few areas of study for which I would argue that inherent talent plays a crucial role. And that is likely more a criticism of mathematics than of the multi-faceted intelligences of humans, as I will discuss more in the conclusion. But I saw such resistance towards Professor Murphy's class in particular as well. Elizabeth, Charlotte, and Markus are a trio of perfect examples from the low-income class, working class, and middle class respectively. They were resisting the dominance of false restricted mathematical code, as well as the new elaborated mathematical code they had to navigate to be successful in Professor Murphy's class.

Nevertheless, as a result of the privileged poor status of all low-income, working-class, and middle-class participants, there were also connections to the epistemological themes Anyon found within the middle class. Although the middle-class is often a gray, messy mix between the poles of low-income and elite, *possibility* is so often the central tenet. Participants across social classes treated math as a means to an end, justifying their enrollment in the class as a sort of exchange for further classes, graduation, jobs, status, etc. Moreover, just as Anyon found that middle-class schools prioritized knowledge for daily life, moderately more middle-class and privileged poor participants framed finances as a core value of mathematics. This was still a value for upper-class participants, but they were better able to differentiate such practical applications from the "real math" of school. This "realness" is, of course, fabricated and classed. It is a means for the upper classes to hoard status, as I will dissect in the conclusion.

The last two values that students emphasized were problem-solving and conceptualizing the world. It was difficult to accurately analyze these themes, as the term "problem-solving" has become commodified beyond recognition in mathematics education in the decades since her seminal studies. I had trouble isolating problem-solving from practical application, especially when students mentioned it only briefly. These days, it is easy to talk of valuing problem-solving without actually valuing mathematical reasoning, despite Anyon tying the concepts together as emblematic of executive elite classrooms (as a reminder, she categorized social classes differently than I chose to). Still, it was unsurprising to see Professor Murphy focus so strongly on building mathematical intuition and reasoning abilities in an elite space like Miller. And it was unsurprising to only have an elite student and a middle-class student with a professor as a father mention problem-solving during the first interview. But Anyon would have been very surprised to hear working-class Nazia talk about how the problem-solving nature of her high school and the rather dismissive attitude that elite Prisha had towards problem-solving. It was, more or less, a mixed bag. I suspect, however, that this was due to the dearth of doubly disadvantaged participants and the aforementioned commodification of problem-solving in mathematics education. Nazia's high school experience is a good example of this commodification.

At this point, I should write a bit about Sean. He mentioned problem-solving extensively and in every interview. As one of two elite students in the class, he was the undeniable outlier in attitude towards mathematics. There were special cases among the other social classes, but Sean was remarkable. He simply could not stop talking about the value of problem-solving. It was easy to see his reverence for logical reasoning, just as Anyon found was typical of executive elite education epistemologies, and his comfort with the language of the class surpassed any other participant. Most of the reservations he had about the class were diametrically opposed to his classmates, as he wanted more abstraction and decreased emphasis on grading. Outliers are just that, this study cannot be based upon the interviews of one student. Nevertheless, it is undeniably significant that Sean was of elite social class.

Finally, conceptualizing the world. Anyon observed this valuation of knowledge most strongly in affluent professional schools, which is more or less equivalent to upper-class. This was the valuation of mathematics with the strongest correlation to social class, along with "not valuable." Conceptualizing the world was most prevalent in Mariam, Claire, and Prisha, who are middle-class, upper-class, and elite respectively. They viewed math as a way to alter how they thought about the world and build lenses to understand their experience. This wasn't about finances or logical reasoning, but individual experience, and mirrored Anyon's study of affluent professional schools. In those places, as with upper-class-adjacent participants in this study, knowledge means to creatively make sense of the world around them. As Claire said, even a potter can see new geometrical patterns and symmetries.

And although Mariam and Prisha couldn't put their finger on exactly how mathematics might prove useful in conceptualizing the world, that's exactly the point. For this value, it is nearly impossible to say how it might be useful until it actually is, and even then, they likely won't know they are drawing upon what they learned in mathematics. It is truly creative. It is independent. Those participants coming from low-income and working-class backgrounds did not understand the mathematics of Professor Murphy's class in this way, and this mismatch results in disparate outcomes (beyond grades), including disparate attitudes towards mathematics and disparate levels of effort needed to pass the class.

Conclusion

When I embarked upon this study, trying to find motivations and theoretical frameworks, I had just taken an epistemology class taught within the philosophy department. I was intrigued by the ethical implications of epistemologies and was drawn to Fricker's (2007) ideas of distinctly epistemic injustices. Inherent to this idea is that when epistemic injustice occurs, something is lost. Some way of knowing the world is ignored or pushed aside by hegemonic epistemologies, leaving us with a smaller toolkit for understanding, as well as people whose epistemology has been stripped and/or delegitimized. Yet, this study was not an expansive mathematical ethnography. Discussions of mathematical epistemology were instead somewhat situated in the educational context. Thus, my direct analysis relied upon education academics like Anyon and Lareau more so than epistemological theories and the sociology of mathematics. This mild disconnect between my literature review and analysis was a result of timeframe restrictions that forced me to write the literature review before I had even completed data collection. If I were to do this project over with hindsight, I would have explored the literature around language and status more extensively.

In the end, interviews with limited observations and field notes are no substitute for a large-scale ethnography. While I was able to make the context of Professor Murphy's mathematics class a core element of my study, by the end, I was left yearning for a more comprehensive ethnography of social class and mathematics. Mathematics may hold a special kind of status, but how is it actually used in communities? What effect does it have? How do students grow? Some of these questions are impossible to answer with accuracy that would satisfy the positivist. But that's ok. We can still gain a better grasp of what mathematics means to people. And we need to, because our understanding of mathematics in this way lags behind the other “core” subjects of history, language, the sciences, and the arts. We seem to have a better understanding of what those subjects mean to us as individuals as to our communities, but we are at a loss for math. For some reason, most mathematics as we understand it happens in schools. When thinking about history, science, art, or language, people do not merely think about the high school class they took in the subject as they do with mathematics. That is why students in the study so often didn’t know how to answer my questions, and that is why I had to provide plentiful scaffolding for participants to navigate their relationships with mathematics. There is more ethnomathematical work to be done here.

For now, we can say something meaningful about epistemic virtues that are connected to mathematics education, and thus about epistemic injustice in elite mathematics classrooms as well. Drawing upon Zagzebski’s (1996) virtue epistemology, I posit that nearly all human cultures hold the epistemic virtue of rational thinking, as reasoning seems to be part of what it means to be human. There are nuances within the epistemic virtue of rationality, however. As elucidated in the literature review, there are mismatches between epistemologies that are ignored and paved over by dominant groups. In the language codes of the low-income/working-class students we saw an epistemic virtue of rationality that was supported by the practical benefits of mathematics. These primarily included mathematics as a means to an end and as a way to figure out finances, to gain credentials to be exchanged for college and jobs (Anyon, 1981), and otherwise as not valuable. A more extensive sociomathematical ethnography is needed to fill out the picture of the low-income/working-class epistemic virtue of rationality and its relation to mathematics. Meanwhile, in the elaborated mathematical code of the upper-class and the ways they conceptualize mathematical knowledge, the upper-class epistemic virtue of rationality is self-justified. That is, elite rationality is justified as a virtue in-and-of itself to a far greater extent than an outsider to the upper-class might expect. In other words, the elite epistemic virtue of rationality is largely self-justified in elite institutions and ways of knowing.

Why might this be? And what might result from this? To answer these questions, we first need to have a conversation about the status around mathematics. Status buttresses the idea of epistemic injustice to provide an answer to the "so what?" missing from the analysis section. After all, it is healthy to have a mix of different conceptions of mathematical knowledge and codes of mathematical language. Every subculture has their own dialects and epistemologies that need to be respected and heard. But in the end, students didn't talk much about mathematical epistemology explicitly. Instead, they talked about grades. It was telling that grades were the backbone of discussions about motivations, as I had more quotes about grades to pull from than any other subtheme. There is more research to be done about the ways in which grading is both socially and institutionally emphasized in mathematics as compared to other subjects, as I suspect students consider grades to be a more essential component of mathematics education.

Critically, participants considered the ability to take and pass an advanced college math class is a status marker in and of itself. Their peers, professors, parents, and potential employers all saw mathematics as a marker of a smart and determined student, as a college math class looks especially shiny on a transcript, even if it is not required for the job. Students internalize these ideas, feeling proud of themselves for surviving the course. This is not what education should be about. On top of that, we must consider who gets access to this status, as in the end, mathematics is a male-dominated field and as we've seen in this study, the mathematics which garners the most status (elite mathematical knowledge and language) is less accessible to low-income and working-class students. Here, I am less concerned with the cases of exceptional natural mathematical ability as I am with the typical student, the student who decided to take an introductory college calculus class even though they weren't planning on being a mathematics major. These are the majority of students. And for these students, status is inequitably distributed. Yet there is a defensiveness around the status upper-class and elite people have accumulated as they maintain traditionally elite ways of understanding the world.

And that brings us to an answer to the question we asked a page ago: why is the epistemic virtue of rationality largely self-justified in elite institutions and elite ways of knowing? Why is mathematics so emphasized in elite educational institutions, as well as educational institutions at large? The answer is connected to the status of mathematics that participants kept talking about. My last semester of college, I was invited to join Phi Beta Kappa, one of the most prestigious academic honors societies. On their website, they list five stipulations for membership, one of which as follows:

> Stipulation 4 – The candidate's undergraduate record shall include at least one course in college-level mathematics, logic, or statistics, with content appropriate to a liberal arts and sciences curriculum. The course should introduce the student to mathematical ideas, abstract thinking, proofs, and the axiomatic method (Phi Beta Kappa, 2011).

The elite, self-justifying epistemic virtue of rationality is starkly clear in this stipulation. Mathematics is a traditional discipline at heart, meant to inculcate the logical reasoning central to orthodox Western education. Teachers have been trained in the overemphasized Western metaphysical dualism of the mind and body, of rationality and emotion, in conjunction with an elevation of the mind over the body. As bell hooks (1994) notes, teachers are conditioned to enter the classroom space and teach as if only rational minds are present, not holistic bodies. She perceptively asserts that the body and the passion it brings "can provide an epistemological grounding" (p. 195). Yes, mathematics has astonishingly fruitful applications, but as Professor

Murphy said, that's just a little bit of fluff on the side. To the elite, that's not what mathematics is really about. It is about creating rational thinkers molded after the white male Western philosophers of the past. It is about clinging onto the superiority of rationality to justify their own status. It is about maintaining a pure epistemic virtue of rationality, free from worldly justifications that the low-income, working-class, and even middle class use. That is what the elite is doing.

Admittedly, the mathematics of the working-class and adjacent social classes is not necessarily better. Yes, they may frame mathematics as a tool for daily life more adeptly, but they both over stress the memorization of formulas in a gambit to access dominant cultures and ignore the value of mathematics as a way of understanding the world and nourishing logical reasoning. And yes, I just criticized elite mathematics for focusing too heavily on these two ideals. But the truth can be found in both epistemologies, and the elite is ignoring this violently. They are committing epistemic injustice, specifically epistemic classism, imposing their version of an epistemic virtue of rationality upon society as a whole. As a result, their mathematics has maintained its status above other disciplines.

Plus, in emphasizing the elite, self-justifying epistemic virtue of rationality, elite institutions commit epistemic injustice by erasing mathematics as a tool of daily life, especially for those students that continue their mathematics education. In my own experience as a middle-class undergraduate student at an elite institution, I slowly started to become mathematically elitist. I considered fields such as statistics, mathematical sciences, and applied mathematics as inferior from a mathematical perspective, and stopped valuing their insights as deeply. I lost track of what makes mathematics relevant to the people in my community outside of school, even if part of that may have been due to my independent agency as someone who truly loves mathematics.

From Professor Murphy, to Sean, to Markus's private school, it is ironic that the subtheme of math being a subject for everyone was largely limited to upper-class and elite students and institutions. To them, mathematics was a subject open to all, no lab equipment or novels required. And logical thinking was something everyone could appreciate, regardless of who they were. This has truth in it, as does math's importance in daily life, but it ignores the questions of access and status I just surfaced. Only certain types of mathematical knowledge have status, and only certain types of people get the easiest access to that type of mathematical knowledge. Places like Miller College need to recognize this and work to understand the variety of epistemic fluencies that students of different social classes have upon matriculation. Professor Murphy did well to explicitly tell students how he valued mathematics, but that is just a first step.

I am not claiming that colleges need to synthesize epistemologies, as that has historically resulted in inadvertent subjugation of the marginalized group's epistemologies, but something needs to be done. Colleges must recognize this mismatch and provide pathways for low-income/working-class students to access elite mathematical knowledge and language, all the while cultivating a critical stance toward that knowledge and language. This could look like additional support, more explicit expectations, and conversations around what mathematics means to different people. Amazingly, such conversations are nowhere to be found in mathematics courses. For the participants in my study at Miller College, epistemic classism clearly occurred, as the epistemologies of low-income/working-class/middle-class students didn't match that of the elite college they attended, and that mismatch was plainly ignored.

At the end of the day, I suspect that after future research, we may find mathematics just isn't as intrinsically important as our education systems make it out to be. Or more precisely, mathematics *as currently constructed by our education systems* (systems which are governed by elite epistemic virtues) is overemphasized and unduly imposed. As currently built, I believe mathematics provides among the least value of any standard school subject, largely regardless of social class. It is broken and needs to be renovated, not repaired. Above all, mathematics needs to become more human. This has not happened already due to the defensiveness of the elite as they work to maintain their status using epistemic injustice as their tool.

Keep in mind, I say all of this as a current and future high school mathematics educator that adores mathematics. I savored my classes with my favorite mathematics professors and continue to savor my classes with my students. Plus, this line critique is of great detriment to my own status, as I have a reputation as a skilled mathematical thinker. And it's not unwarranted. My logical reasoning skills are superb. But the educational system has overemphasized rationality through the self-justifying primacy of mathematics, de-emphasizing the importance of creativity and other intelligences. For example, in mathematics itself, creativity is undervalued until well into one's undergraduate studies. I am not an exceptionally creative person in mathematics or in any subject. I can combine and synthesize ideas, drawing from seemingly unrelated ideas and creating new ones, yet I rarely create something from nothing, as even the seeds of my songwriting can always be found in other songs. While this sort of creation is commonplace and likely the norm, I can still admit that I am distinctly inept at something-from-nothing creativity. And that my writing is rather dry. But that's ok. The problem is that people like me are unjustifiably labeled as more "intelligent" than others, accepted into more prestigious schools, seen as more virtuous, and attain higher social status (as a side note, this is one of the reasons I am hesitant to venture into the world of academia). In addition, there are issues of sexism, racism, and classism at play, as people who are labeled as "intelligent" tend to look, act, and speak a certain way. And all of these forces surely helped create and sustain the massive gender gap in mathematics. Sadly, all of that is for another paper.

I will leave you with some considerations following Dewey's (1916) sentiment that education is not mere preparation for life, education is life itself. In Dewey's work, education and life are not abstractions, but deeply social endeavors that we engage with as whole humans. They are a social life. Then how much of mathematics education as currently constructed is life itself as compared to history, language, the sciences, and the arts? And if we conclude that it plays a lesser or equal role in life itself as compared to those other disciplines, then why does mathematics hold a special status among disciplines? Why is an elite self-justifying epistemic virtue of rationality so emphasized in schools? A question well put is a question half answered, so I urge you to continue uncovering and dissecting these questions. Rephrase them, recapitulate them, test hypotheses, and bring in myriad perspectives I left out, such as the political and economic. And only then, begin to answer them. I look forward to hearing your voices.

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Appendix

Interview Scripts

Interview 1 Script:

- What high school did you attend?
    - Is the high school you attended located in the community that you spent the majority of your childhood?
        1. *If not, where was the community you spent the most of your childhood?
- Outside of the school setting, how can you tell which adults are good at math in your home community?
    - In what settings is mathematical ability showcased most often and prominently in your home community?
    - Can you give a specific example of an adult who you consider good at mathematics in your home community?
        1. What instance(s) showcased this?
    - Can you give a specific example of an adult who you consider bad at mathematics in your home community?
        1. What instance(s) showcased this?
    - What was the ultimate test of mathematical knowledge and understanding for adults in your home community?
- How could you tell which of your peers were good at mathematics in the school setting in your home community?
    - Can you give a specific example of a peer in school who you considered to be good at mathematics in your home community?

        i. What instance(s) showcased this?

    - Can you give a specific example of a peer who you considered to be bad at mathematics in school in your high school?
        i. What instance(s) showcased this?
    - In what situations was mathematical knowledge and understanding showcased most often and prominently in school your home community?
    - What was the ultimate test of mathematical knowledge and understanding for you and your peers in the school setting in your home community?
- Outside of the school setting, how can you tell if an answer to a math-related question is correct in your home community?
    - Who or what makes this decision?
    - Can you give a specific instance of when a math-related answer was or would be considered correct outside of school in your home community?
    - Can you give a specific instance of when a math-related answer was or would be considered incorrect outside of school in your home community?
- In the school setting, how could you tell if an answer to a math-related question was correct in your community?
    - Who or what makes this decision?
    - Can you give a specific instance of when a math-related answer was or would be considered correct in the school setting?
    - Can you give a specific instance of when a math-related answer was or would be considered incorrect in the school setting?

- In the school setting, what did the day-to-day process of doing homework look like in your home community?
  - Did you typically work on homework alone?
    i. Why?
  - Did you typically consult anyone for guidance?
    i. Why?
- Outside of the school setting, to who or to what would you go if you were stuck on a math-related problem in your home community?
  - Why?
  - Can you give a specific example of this?
    i. Was this the typical resource for such a problem for others in your community?
    ii. *If not, why? And what was?
- In the school setting, to who or to what would you go if you were stuck on a math-related problem in your home community?
  - Why?
  - Can you give a specific example of this?
    i. Was this the typical resource for such a problem for others in your high school?
    ii. *If not, why? And what was?
- Outside of the school setting, who or what is your most trusted mathematics resource in your home community?
  - Why?

- In the school setting, who or what was your most trusted mathematics resource in your home community?
    - Why?
- Outside of the school setting, when is math most valuable to people in your home community?
    - Why was that situation especially valuable? What gave it its value?
- In the school setting, when was math most valuable to you and your peers in your home community?
    - Why was that situation especially valuable? What gave it its value?
- In your home community outside of school, what is math?
    - How would you describe the role of math in your home community to someone at Miller?
- In your home community in the school setting, what is math?
    - How would you describe/conceptualize the role of math in your high school to someone Miller?

Interview 2 Script:

- In this class, how can you tell which students are good at math?
    - Can you give a specific example of a peer who you consider good at mathematics in this class?
        i. What instance(s) showcased this?
    - Can you give a specific example of a peer who you consider bad at mathematics in this class?
        i. What instance(s) showcased this?

- Outside of mathematics classes, how can you tell which students are good at math at Miller?
    - Can you give a specific example of a peer who you consider good at mathematics that you are not aware has taken a math class at Miller?
        i. What instance(s) showcased this?
    - Can you give a specific example of a peer who you consider bad at mathematics that you are not aware has taken a math class at Miller?
        i. What instance(s) showcased this?
- In this class, how can you tell if an answer to a math question is correct?
    - Who or what makes that decision?
- Outside of mathematics classes, how can you tell if an answer to a math-related question is correct at Miller?
    - Who or what makes this decision?
- In this class, what does the day-to-day process of doing homework look like?
    - Do you typically work on homework alone?
        i. Why?
    - Do you typically consult anyone for guidance?
        i. Why?
- Familiarity vs. Foreign
    - How familiar is the structure of class time?
    - How familiar is the structure and process of homework?
    - How familiar is the structure and type of questions on exams?
        i. How familiar are the types of questions on hw and exams?

  - How much of your thought process (your work) are you expected to show for your answers on homework and exams?
    i. Is it more or less than you did in high school?
  - What do you think of the latex requirement?
    i. Why does Professor Murphy do this?
  - What do you think of your experience with TA hours?
  - What is most familiar?
  - What is least familiar?
  -
- To who or to what would you go if you are stuck on a math problem for this class?
  - Why?
- Still at Miller, to who or to what would you go if you are stuck on a math-related problem *not* for this class?
  - Why?
- For this class, who or what is your most trusted mathematics resource?
  - Why?
- Outside of mathematics classes, who or what is your most trusted mathematics resource at Miller?
  - Why?
- In the context of this class, when is math most valuable to you and your peers?
  - Why was that situation especially valuable? What gave it its value?
- Outside of mathematics classes, when is math most valuable to you and your peers at Miller?

    - Why was that situation especially valuable? What gave it its value?
- In this class, what is math?
    - How would you describe/conceptualize the role of math in this class to someone back home?
- Outside of mathematics classes, what is math to the people at Miller?
    - How would you describe/conceptualize the role of math at Miller outside of mathematics classes to someone back home?

Interview 3 Script:

- Why did you sign up for this course?
- Now that the class is finished, did you find the course as a whole a beneficial learning experience? Do you feel like you learned something meaningful in the course?
- Do you plan to take any more math classes at Miller?
    - If not, why?
        - How will what you learned in this course help (or not help) in future non-mathematics classes?
    - If so, which classes/what type of classes?
        - Why (or why not) do you plan to take these future math courses?
        - What do you hope to learn in these future math courses?
- Did this class affect your plan for your academic journey at Miller?
    - If so, how?
        - What new classes are you planning on taking?
        - What classes are you no longer planning on taking?
    - Or why not?

- Did this class affect your plan for your career after Miller?
    - If so, how?
        - What new career paths did this class make you consider, if it did?
        - What career paths did this class close off to you, if it did?
    - Or why not?
- Did this class affect how you think about mathematics?
    - If so, how?
    - Or why not?
    - Is this similar or different to how you thought about math previously, in high school or otherwise?
- Did this class affect how you think about learning in general?
    - If so, how?
    - Or why not?
    - Is this similar or different to how you thought about learning previously, in high school or otherwise?
- Now that you can look back over your experience, what do you feel like you learned from this course and from Professor Murphy? What knowledge did you gain?
    - Is there anything else? (I expect needing to draw out answers)
    - How will what you learned in this course help (or not help) in future math classes?
    - How will what you learned in this course help (or not help) in future non-mathematics classes?

- ○ How will what you learned in this class be a part of the rest of your life after college, if it will be?

Professor Interview Script:

The interview with the professor drew on questions from all three student interview scripts.

Debriefing Script:

- The focus of my study is how the norms of college mathematics classrooms interact with social class variations within the student body. Have any of your views changed about yourself, others, mathematics, or the world over the course of the observation and interview process?
- Were there any situations during the research project that challenged you or questions difficult for you to answer? If so, why?
- What was your favorite part of the observation and interview process? Your least favorite?
- Are there any questions you wish that I had asked you, and would like to answer now?
- Is there anything more you wish to share?