2002

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Alison Culpen
Colby College

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An Analysis of the Colby Quarterly Econometric Model of the U.S. Economy Using the Economic Effects of September 11th
By Alison M. Culpen

The economic repercussions of September 11th are unique in that never before have economists needed to forecast and examine the impact of an event of such magnitude. This paper explains many of the economic effects of September 11th, from the initial aftermath to recent developments. These effects are used to analyze the Colby Quarterly Econometric Model of the U.S. economy by simulating the model with actions taken by the government, such as an increase in spending, as well as other economic consequences resulting from September 11th. These simulations are then examined in more detail through multipliers that ultimately reveal the theoretical properties of the Colby model, which is found to differ from other models of similar nature. This paper contributes to the literature on macroeconometric models by further examining their uses and limitations through a recent string of economic repercussions resulting from September 11th.

Colby College, May 2002

Alison M. Culpen is currently an Economics student at Colby College. She thanks Professor Michael R. Donihue and Professor Randy A. Nelson for all of their help and encouragement.
Section I: Introduction

September 11th was a tragic day for the United States of America. From an emotional perspective, it was a day of horror, anger, and sadness. From an economic perspective, it was a day that spawned doubt in the government and financial systems, and provoked even more uncertainty about the lingering possibility of a recession. In the midst of grieving for people who had lost loved ones and thinking about who could do such a horrible act and why, there were a few other concerns on everyone's minds. What would the economic repercussions be? With economic activity already declining since March, would these events push the economy into a recession? If so, what would be the driving force behind the recession; would business and consumer confidence be driven down to record levels? Would the stock market tank and ever climb back to its previous level?

Although many months later, we do have some of these answers, we still do not know how these events will change our world in the long run. Large macroeconometric models have been built to examine the economy and give us more insight into some of the hovering questions about how different occurrences can change our economic conditions in the short and long run. Although these models have been scrutinized by many economic scholars who claim them to be ineffective in the presence of changing conditions, there are a number of ways these models can be utilized.

The Senior Seminar in Economic Forecasting at Colby College, led by Professor Michael Donihue, decided to take on the task of building Colby's own macroeconometric model of the U.S. economy, the Colby Quarterly Econometric Model (CQEM). This model could examine in more detail how the events of September 11th would take its toll on the economy through numerous forecasts of various sectors and of GDP as a whole. Never before had such an event
occurred, and forecasting a model into the future in the wake of the circumstances was important—especially with even Alan Greenspan saying, "nobody has the capacity to fathom fully how the effects of the tragedy of September 11th will play out in our economy". However, as with all econometric models, there is always a concern with the accuracy of the predictions and the model in general. Was it precise and thorough enough to make these important predictions? To answer this question, this study goes further by examining the structural properties of the model and comparing it with others of similar nature.

Using the CQEM, this paper looks at shocks to the model that incorporate some of the effects of September 11th, and examines the properties of the model—through dynamic multipliers—to see how the model reacts to such changes. Tests are then performed on the CQEM similar to those done on other macroeconometric models to examine its comparative properties. Through these shocks, this study aims to uncover the properties of the CQEM while contributing to the literature on macroeconometric models by further examining their uses and limitations. In order to explain the shocks to the model, however, the economic effects of September 11th should first be looked at in more detail.

**Section II: What We Know About September 11th to Date**

**Initial Economic Effects** As we know from past disasters, markets do not react well to bad news, let alone terrorist attacks on the hub of the financial community and government capital. Thus, it should be noted how quickly the Federal Reserve (Fed) responded to the events of September 11th to prevent as much shock to the markets as possible. On September 12th, a press statement was made saying the Fed was operating and that the discount window was available to

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1 Doward, 3/10/01.
meet liquidity needs. Over the next three days, the Fed injected a total amount of over $100 billion of liquidity through discount window loans and open market operations. It also let the federal funds rate (the interest rate banks charge each other for overnight loans) slip below the target rate for a few days to ensure ample liquidity especially abroad, and in the following two weeks lowered the federal funds rate twice by one hundred basis points in total. This helped immensely, and prevented the stock market from crashing, although it did drop significantly the day it opened. The low for the Dow Jones Industrial Average (Dow) was September 19th when it stood at a mere 8,480—12% below its September 10th close.

Like the Fed, Congress also responded quickly. It passed a $40 billion aid package for increased domestic security and military spending soon after the attacks, with another $15 billion approved for the airline industry. The Federal government also did a good job of assuring U.S. citizens that it would do as much as it could in terms of increased security measures and finding the people responsible for the terrorist attacks.

From September to October, the unemployment rate jumped from 5.0% to 5.4%. Most categories of consumer spending plummeted in September, but recouped a bit in October due in part to the zero percent financing deals the auto industry implemented and the low price of oil at that time. Industrial and manufacturing output as well as business spending dropped sharply following the attacks, though all three had been declining for some time. Finally, real GDP in the third quarter contracted by half of a percentage point due to the extreme economic downturn following the attacks.

The airline industry was hit hard in the months after September 11th, even with an aid package in the making. In October, business travel fell 40%, and Boeing itself cut more than

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2 This paragraph is referenced from Parry, 12/7/01.
3 Koch, 10/01.
5,000 workers.\textsuperscript{4} Orders for new aircraft were cancelled as people became scared of flying, though rates went down to attract more travelers.

President Bush and the Administration declared war on terrorism globally and began attacking terrorist camps in Afghanistan by bombing them. The usual wartime economy this time is not so usual, but does allow for more government spending, though most aid is needed to reconstruct and support the home turf. Military spending during a war can add up to as much as 60-70\% of pre-war GDP. This time, however, a lot of spending is needed in New York, Washington, and to bail out the airline and insurance industries not to mention funding the war abroad. Offsetting this are all of the losses resulting from increased fears of flying and of another attack, lowering demand and investment measures in many areas of the economy. This makes for a different wartime economy than we are used to.

**The Last Six Months** With consumption equal to two thirds of GDP, consumers are an integral part of the health of the economy. Consumer spending has risen almost every month since its decline in September, and although consumer confidence has bounced around, it is generally heading upward. The tax refunds from early last year have made a difference ($117 billion in total to individuals), combined with low oil prices, zero percent financing deals, and mortgage refinancing (due to successive interest rate decreases by the Fed). All of these positive spending effects seem to have partially cancelled out the negative effects associated with increasing unemployment—especially in the manufacturing sector, which experienced the most layoffs.

Although the Dow has bounced around due to the Enron scandal as well as mixed economic news and earnings releases, it has surpassed pre-attack levels. Corporate profits and

\textsuperscript{4} Doward, 3/10/01.
spending have been down, with debt on the high side; investors are skeptical of recent earnings projections, and should be. A few major companies recently announced first quarter losses including IBM, Ford, Boeing (net loss of 1.25 billion), and General Electric (net income down 2.7%). However, CitiGroup, Intel, and Microsoft have all reported increases in earnings, creating even more volatility in the markets.

Despite payrolls increasing this first quarter mainly in the service sector, the unemployment rate jumped back up to 5.7% in March from its first drop in months to 5.5% in February—still well above its pre-recession low of 3.9%. Temporary staffing services used by businesses to provide flexibility has contributed to this rate because the workers are not permanent.

In mid-March, the Administration signed a bill for $51 billion dollars (revised from initial estimates of $80-$100 billion) in tax relief for businesses in lower Manhattan, as well as extended unemployment benefits to jobless workers and tax incentives for investment in lower Manhattan. The bill also is designed to pump an additional $94 billion into the economy over the next five years. It took six months for Democrats and Republicans to agree over the bill—government spending has risen 10% above pre-September 11th plans. The issue of whether or not this anti-recession bill was needed so late in the process is still controversial, however it should help speed up the recovery. There are also a few more forms of supplemental budget proposals and aid packages under negotiation to fund the War on Terrorism as well as other jobless workers. Recently President Bush proposed a $379 billion budget for 2003 and says he

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5 The following statistics are referenced from the Wall Street Journal.
6 The following two paragraphs are referenced from Ip, 4/8/02.
7 Murray and McKinnon, 3/11/02.
wants to make sure increases in spending due to the war are partially offset by less spending in other areas of the government to avoid a period of high inflation.\textsuperscript{8}

The Fed has played a huge role in the recovery process, lowering the federal funds rate successively from 5.5\% in February of 2001 to the lowest it has been in four decades, 1.75\% in December. The Fed has not moved rates since then, seeing that inflation is relatively low and productivity still high. This could mean good things for businesses and consumers and, again, is a wise move for the Fed as long as inflation is under control—the CPI has risen by less than 2\% over the past year.\textsuperscript{9} Low interest rates have had a huge effect on the economy, from mortgage refinancing to business investment, to decreasing the savings rate, thus contributing to consumer spending and increasing consumer sentiment. Fed Chairman Alan Greenspan has just announced that rates probably won’t move for a few months, another sign that this should continue. Many are also saying that Greenspan would have a larger margin of error in terms of future rate hikes which will eventually be necessary if he had more help from the fiscal side, maybe in the form of more permanent marginal rate tax cuts.

The most recent news, however, has been that the economy actually grew at an annualized rate of 5.8\% this first quarter, up from 1.7\% in the fourth quarter of 2001.\textsuperscript{10} Most of this increase, however, is due to inventory re-building and should not be looked at as a strong comeback in demand. A decrease in durable goods orders, consumer sentiment, and new home sales solidifies this doubt in the strong GDP data. There has been a lot of mixed economic activity lately—the Fed’s beige book says the recovery process is coming in fits and starts. Everyone is hopeful that business spending will follow consumer spending which just increased; capital spending levels have reduced by less this first quarter than in the fourth quarter of 2001.

\textsuperscript{8} Cummings, 4/17/02.
\textsuperscript{9} Feldstein, 3/13/02.
\textsuperscript{10} The following two paragraphs are referenced from Ip and Caffrey, 4/29/02.
which is a good sign. However, business spending usually does not take off until after profits start recovering, something that has not been happening lately. While the recovery seems to be moving slowly, some say the reason for this is that we didn’t have much of a recession: a few main indicators have been positive this last year such as retail sales and personal income. Productivity has become very important in this “new economy” as well, and some economists say that the most important aspect of economic growth nowadays has been the low prices of information technology.

With the U.S. in an economic slump this past year, the rest of the world has been following suit. Weaker economies such as Germany, Italy and Japan have had outright recessions in the last quarter of 2001, with other nations, such as the UK and much of Europe suffering close behind. The UK just had its lowest first quarter annual rate of growth since 1992, at 1%. Lately, however, as the U.S. economy appears to be gaining some steam, Europe’s unemployment rate seems to be slowing and Germany and even Japan’s economies appear to be making headway after officially declaring a recession at the end of 2001 with GDP shrinking for two consecutive quarters.

Six months after the attacks, the airline industry still faces challenges, but things are improving. As fears of flying diminish, a more constant stream of revenue is coming in and airlines can add flights that they had to drop and make some other necessary upkeep changes. The problem still remains that not as many business customers are flying, and that the fares they have to offer travelers to attract them to fly are often low and bad for profitability due to the high fixed costs associated with the industry. Continental Airlines just announced a loss of $166 million and it is estimated that collectively the airline industry will report a loss close to $2 billion. Some people are still deterred from flying due to the long security lines that have come
about from an increase in security measures. Labor costs also continue to rise, however an upside is that oil prices are still generally low. Despite the lack of profit, the airline industry has come a long way since September 11th.

Did September 11th Prompt a Recession? Most economists agree that the United States economy was already sluggish before the attacks on the World Trade Center and the Pentagon in September. The technology bubble bursting and the subsequent drop in equity values, business investment and manufacturing output were all major contributions to the economic slow-down pre-September 11th. Although growth was still in the positive range (just barely), consumers were holding things together with their long-run optimism and positive prospects about the future.

However, the massive layoffs, shocks to consumer confidence and spending (the economy’s main support), and the large drop in the stock market post September 11th may have pushed the economy over the edge in the months following the attacks—this is still a controversial issue. Although in the beginning most layoffs were in the travel and tourism industries, since the end of last year they have spread (especially in manufacturing) as can be seen in the unemployment rate rising this March after a brief decline. It is generally agreed that the increase in the unemployment rate by 1.5 percentage points between March and December of 2001 is what prompted the National Bureau of Economic Research (NBER) to decide last November that a recession began over a year ago. However, by the usual standards of economic output (the volume of goods and services produced), there may never have been a recession. GDP shrank for only one quarter, and not by a lot; the reason for this is that unlike most recessions where productivity declines, last year the economy was able to produce more

11 The following section is referenced from Ip, 3/11/02.
with less which is very rare during a period of economic downturn—productivity increased. With the NBER's definition of a recession as a few months of significant economic decline, it is now hard to define economic activity: is it input such as employment, or output—GDP? Never before has the NBER had to think so much about how it has to define economic activity and claims that it still could change its decision on the recession but has no inclination to do so in the near future. The NBER also looks at production and retail sales, which did fall, as well as personal disposable income (inflation adjusted), which rose. However, it states that unemployment is its only consistent indicator of recessions. GDP, because it is constantly revised, statistically estimated, and is a quarterly statistic, is not used as a criterion. Many other economists beg to differ on this decision, however, and look not only at GDP as an indicator, but also how people are feeling about the economy in general. Despite some good news, most would agree the picture is still a bit bleak. This is evident in the consumer sentiment index, which has dropped recently.

With the economic effects of September 11th now explained in more detail, it is appropriate to next talk about how to model the macroeconomy through econometric models.

Section III: The NIPA, the Multiplier, and Econometric Models

The national income and product accounts (NIPA) are the main source of data used by macroeconomists. The NIPA describe the components of national income and output in the economy and are produced by the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce. The NIPA also provide macroeconomists with a conceptual framework for how the economy should fit together. Gross domestic product (GDP - the total market value of a country's output) is the key component of this accounting framework, and is equal to

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12 The following section is referenced from Case and Fair (2002).
consumption plus investment plus government spending plus exports minus imports (GDP = C + I + G + X - M). This is known as the expenditure approach to calculating GDP since all of the above components are classified as spending.

Along with the NIPA, the multiplier process is another very important concept in macroeconomics. The multiplier, as defined by Case and Fair, is "the ratio of the change in the equilibrium level of output to a change in some autonomous variable."13 Suppose, for example, government spending increases by $40 billion, the change in equilibrium output will be more than $40 billion. This is the multiplier effect, and has important implications on the macroeconomy. When government spending is increased (a component of aggregate spending), planned spending will increase and be greater than output—inventory will be lower than planned, and firms will have an incentive to increase output. As output rises the economy is generating more income, firms need to hire new workers who then spend some of their income. Higher consumption spending leads to planned spending being higher than output again. Inventories are lower than planned; firms raise output which increases income—the process starts again. The multiplier process stops once the effects of the increase in government spending dampens out, as will be examined in Section X of this paper. As this study begins to examine economic models that are created to model the macroeconomy in the best way possible, the NIPA and multiplier process become important factors. As is found, modeling economies is a very complex and difficult task and is not in any way an exact science.

An article by Sowey (1987) defines econometrics as the following, "the discipline in which one studies the theoretical and practical aspects of applying statistical methods to

economic data for the purpose of testing economic theories (represented by carefully constructed models) and of forecasting and controlling the future path of economic variables. Econometric models of the economy came into use in the 1930's and became very helpful in explaining the theory of output and unemployment by J.M. Keynes. From there, models progressed to being statistically estimated and were used to examine conditions of the real world. Most models today contain the basics of Keynesian theory along with many other complex issues which take into account high degrees of detail and sophistication to make them realistic. Models have become the major tool of economic analysis for the private and public sector; governments and businesses use models to generate information and make predictions about many important decisions.

In the early 1970's, economists were using large models of the U.S. economy to analyze and forecast the effects of alternative government policies on the economy. These models were widely used and thought to be very effective. They were based on the theory that consumers and businesses did not adapt their behavior to different government policies, and that every policy the government implemented would have a predictable result. These assumptions were all derived from Keynesian economics, the economic theory that active government intervention and monetary policy leads to economic growth and stability.

**Section IV: Robert Lucas' Contributions to Econometric Modeling**

Enter Robert Lucas and his theory about rational expectations. First devised as an assumption by John Martin in 1961, Lucas investigated this concept and its implications for

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15 The following section is referenced from Klein and Young (1980).
economic policy. Lucas' theory of rational expectations showed that the expectations of consumers and businesses change when the government alters its policy; therefore predictions about the outcomes of the government's actions would have to be revised. Although this idea was not a new one, Lucas was the first to apply this idea in the context of analyzing econometric models, the history behind them, and suggesting a new way to look at the effects of governmental policy.

There are two main parts to rational expectations as explained by the Chicago School of Economics in the critique entitled “Robert Lucas and Rational Expectations”. Lucas examined the old concept that recessions are self-correcting. That is, people generally recognize their own hardships before those of others, but once they do recognize the signs of a recession, the market will take steps to recover from it. Producers will cut prices to attract customers, and workers will lower their wage demand to attract work; the economy in this sense self-corrects in the same way an increase in the money supply would. The government should simply wait out the recession and not intervene at all.

The second part to the rational expectations theory is that government intervention can only range from ineffectiveness to harmful. That is, Lucas suggested that the Fed is no quicker than the markets in acknowledging a recession. The Fed, just like businessmen, uses economic indicators to determine the state of the economy. Hence, if both parties are looking at the same indicators, then a Fed increase in the money supply will be ineffective if producers are already lowering prices.

Lucas then supported more of Milton Friedman's argument that if the Fed adjusts the money supply by one percent every time the unemployment rate increases by one percent (to

* An increase in the amount of money and credit circulating in the economy, affected by actions of the Fed such as conducting an open market operation. In this case, buying securities (usually government securities) to increase the number of reserves a depository institution holds and thus can loan out.)
create a predictable anti-recession policy), then rational businessmen will grow to expect this and monetary policy is ineffective. That is, these "rational expectations" would make an automatic policy response, and so in order for monetary policy to be effective, the Fed would have to surprise people with random increases and decreases in the money supply. This, however, can make the economy unstable and we can conclude that this effort by the government to control the economy can lead to more harm than good.

Despite Lucas's complex mathematical discussion of this theory, there are some flaws to rational expectations. According to Lucas, "The implicit presumption in these Keynesian models was that people could be fooled over and over again."16 The main assumptions in Lucas' theory, however, are that businessmen regularly keep up with macroeconomic trends and that humans are always perfectly rational and are always perfectly informed. As we can note today, most people are only close to rational and close to being fully informed; this makes a big difference. Most producers will only lower their prices if they have to, and most workers will only lower their wages if they are forced to through market conditions. Therefore, Keynesian economics is effective, and the government and the Fed can intervene to do something in a recession. It's a good thing this works in theory since this past year the Fed has been rigorously trying to revive a sluggish economy using many of its monetary policy tools. Although economists today incorporate some of Lucas' insights into their models of accurately forecasting the effects of alternative government policies, most economic models still have a Keynesian foundation.


16 qtd. in "Robert Lucas and Rational Expectations".
"that estimated parameters that were previously regarded as structural in econometric analysis of economic policy actually depend on the economic policy pursued at the time." For example, the slope of the Phillips curve may depend on fluctuations in money demand and supply at a specific period in time. In this way, the parameters may change with changes in policy; the effects of these policies will be different if people's expectations adjust to them or not. Thus, it is important to take into account people's expectations about future policies when deciding what policies to implement—whether it be the government deciding about a tax reform or the Fed deciding what stance they should have in terms of monetary policy. A normal IS-LM diagram does not do this, it takes expectations as exogenous. If expectations are rational, as Lucas points out, then when the Fed increases the money supply, output will only go up only if people are not expecting this policy to occur; if they do expect it, then the only thing that will go up are prices. Lucas then takes his rational expectations theory a bit further.

Continuing with the analysis of the Royal Swedish Academy of Sciences (1995), Keynesian economists went about making policy decisions using past data to estimate behavioral reduced form functions. They did this because the variables in the structural equations were correlated with one another which would make estimates of the parameters biased. They then made conditional forecasts of the economy with different policies by plugging policy values into the estimated regression equations and solving for the economy's endogenous variables.

There were, however, many problems with this approach that most Keynesian economists were aware of. One is the identification problem. Because their parameters were reduced forms of the structural variables, and policy changes are shown through structural parameters, the estimates using past data were only valid for the old structural parameters. Thus, the estimated parameter values were the wrong values for forecasting the new policy. This problem could be
alleviated by making restrictions on the structural parameter values, however, there would be no basis for these restrictions. As the Lucas critique points out, for the structural equations that represent people's or firm's decisions, their structural parameters change when policy changes. Either way, you get the wrong conditional forecast.

Lucas sums his main point by saying, "I shall argue that the features which lead to success in short-term forecasting are unrelated to quantitative policy evaluation, that the major econometric models are (well) designed to perform the former task only, and that simulations using these models can, in principle, provide no useful information as to the actual consequences of alternative economic policies".¹⁷

Although there are arguments against Lucas's critique today, there have been moves to incorporate expectations and alleviate the problems stated above (VAR models). Econometric models are still used to design policies, as well as short-term forecasting, both of which are important in examining today's economy.

Section V: Economic Forecasting Using Models

Although Hick's IS-LM curves and the NIPA lay the groundwork for understanding important macroeconomic principles, this approach is not used in present-day forecasting due to the fact that models need more detail and complexity to accurately produce forecasts. This is seen in the bigger forecasting models, like the CQEM, where there is not just consumption, but durable and non-durable consumption, as well as services; there are also subparts to all of these different equations. In contrast with forecasting using a time series approach, where you are looking at past data and random disturbances (shocks) for insight into the future, modeling allows and shows interrelationships between data and variables—for big models, these

¹⁷ Lucas (1976), pp.20.
relationships are almost infinite. Also unlike time series, models have to be solved simultaneously. As Klein and Young point out, "In terms of Wharton models of the U.S. economy, this means solving for nearly 800 variables for the Wharton quarterly model and nearly 1,000 for the Wharton annual model".\footnote{Klein and Young (1980), pp.8.} Although the CQEM is not this large, there are still close to fifty equations that were simulated, with over 200 variables used. Part of the process of building a model like this one is first deciding if all of this effort is worth it. When trying to forecast a short-term interest rate, for example, it may be decided to use a single-equation regression if the benefits of building a huge model don’t outweigh the costs. Fortunately, technology has provided us with the tools necessary to press a button and have a large model solved; if this were not the case, these models would not be as frequently used. Building them, however, is still extremely time and thought consuming.

One advantage of using models is their ability to incorporate external factors (such as monetary reserves, the discount rate—anything that can change with policy) which are measured; their effects on the economy are then estimated in numerical terms. Unlike Lucas, Klein and Young point out that these models can be used to study alternative policies due to their quantitative framework.

Another advantage of models is their ability to incorporate indirect effects into a decision, which again, is a product of the interrelated system of the model. Most economists are familiar with direct effects, such as an increase in income will increase consumption. But what about consumer sentiment or interest rates? How do these variables affect consumption, if at all? These are some of the questions that simultaneous equations allow us to answer. Models also enable us to make more long-run projections than a time series analysis would due to their complex and sophisticated nature. It is no wonder the model approach is so popular today with
examining and analyzing a number of things in detail. This higher form of economic thinking and analysis is very important if we even want to attempt to model our world today.

Section VI: A Keynesian Model and the Simulation Process

According to Klein and Young, "A Keynesian model is a particular example of a theoretical form which can be used for specifying operational definitions for the aggregate measures we need to summarize the wealth of data generated by economic activity".19 They then go on to look at an example of a simple closed-economy Keynesian model of the following form:

\[ Y = C + I + G \]  \hspace{1cm} (1)
\[ C = a_0 + a_1 Y \]  \hspace{1cm} (2)
\[ I = b_0 + b_1 r + b_2 \Delta Y \]  \hspace{1cm} (3)

where:

- \( Y \): income (gross national product)
- \( C \): consumption
- \( I \): investment
- \( G \): government purchases
- \( r \): interest rate
- \( \Delta Y = Y_t - Y_{t-1} \): change in \( Y \)

It should be noted that this is just an extension of the IS curve: if \( S \) = saving,

\[ S = Y - C = I + G \]
\[ S = Y - (a_0 + a_1 Y) \]

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\[ I + G = b_0 + b_1 r + b_2 \Delta Y + G \]

Then to construct the IS curve requires tracing out the points where \( I + G = S \), or solve for values of \( r \) and \( Y \) which satisfy the following for any given values of \( G \) and \( Y_{t-1} \):

\[ b_0 + b_1 r + b_2 \Delta Y + G = Y - (a_0 + a_1 Y) \]

It is possible to see if your simulated model satisfies this condition, although it is a difficult task to prove this.

Simulation, according to Pindyck and Rubinfeld (1998), is "the mathematical solution to a simultaneous set of difference equations, which relate the current value of one variable to current and past values of other variables".\(^{20}\) A simulation model refers to this set of equations. Pindyck and Rubinfeld explain the simulation process as follows:

If values are given for the parameters \( a_1, b_1, \) and \( b_2 \), initial values are specified for the variables \( C \) and \( I \), and a time path is given for the exogenous variable \( G \), then the simultaneous solution of these three equations will give us time paths for each of the endogenous variables, \( C \), \( I \), and \( Y \). This is what is meant by the simulation process. Given a model whose parameters have been estimated (or its numerical values otherwise supplied), given initial values for the endogenous variables (i.e. base-year values), and given a time series for the exogenous variables (this may be a historical series or may represent hypotheses about the future behavior of the series), the model is solved over some range of time to yield values for the endogenous variables.\(^{21}\)

Using Klein and Young's equations, one can solve the model by substituting equations (2) and (3) into equation (1) to get:

\[ Y = a_0 + a_1 Y + b_0 + b_1 r + b_2 \Delta Y + G \]

\(^{20}\) Pindyck and Rubinfeld (1998), pp. 381.

\(^{21}\) Pindyck and Rubinfeld (1998), pp. 381.
rearranging:

\[ Y - a_1 Y - b_2 \Delta Y = (a_0 + b_0) + b_1 r + G \]

The solution of this second-order difference equation will depend on initial conditions as well as future values of the exogenous variables (G). This is essentially what a computer program, such as EViews, will do to solve very large models.

The "fit" of the equations, or a measure of how well they explain the data, is as important as it is in single-regressions, but in a different way. In these types of multiple-equation models that go into a larger model, the equations may have a good fit, and yet not reproduce historical data very well when testing the model as a whole. This is where good judgment on the part of the forecaster comes in, and poses one of the big challenges for forecasters.

Forecasting a simulated model involves projecting the model out into the future, beyond the estimation period. Before the forecast is made for the model, it is necessary to have values of each exogenous variable over the forecast horizon. To make sure these forecasts are accurate, one can use ex-post forecasting, or forecasting the variables over a period when we already know what the data are, to see the preciseness of the forecast. The final forecast will be ex-ante, or into the future time period specified, and will be used for predictive purposes.

Section VII: A Simulated Consumption Model

The purpose of this section is to walk through a piece of the process of building a model and simulating it using a specific sector of the CQEM. In the technical appendix of this paper are a number of sections that will allow a thorough explanation of the consumption sector as well as the simulation of this sector. There is first a list of abbreviations for the variables in the CQEM which will be helpful to the reader as many of the variables are used in the equations and
in the text. Next there is a flowchart of the consumption sector which helps to clarify the interrelationships of the sector. The next section includes the equations for the consumption sector followed by a brief explanation of the theory behind the equations. Following this are graphs that will be talked about in this section, and the last part of the appendix shows an example of the source text from the simulation which lists all of the identities of the model, and then each equation is broken down into its components as they are in the model to bring it all together.

The simulation about to be described is only of the consumption sector; that is, holding everything else constant (investment, government spending, etc) this model shows us how well real GDP (GDPH) can be tracked with only consumption as a guide.

Consumption is broken up into durables, non-durables, and services. Durables are things like refrigerators and cars—big-ticket items. Non-durables are things like perishable items—food, clothes, etc. Services are items such as entertainment, travel, and having a stockbroker. What the consumption sector tries to do is model these three parts of consumption in the best way possible. Other variables that go into consumption are consumer sentiment, net worth, price deflators, interest rates, and income.

As can be seen from the graph of real GDP (Graph 1), the model tracked better in earlier years (the simulation started in 1993 and is represented by the dotted line). The computer program, EViews, solved the model in the way described previously; it combined all of the equations of the three parts of consumption—durable, non-durable and services—with the price deflators, and using an identity for real GDP and nominal GDP, it held everything else exogenous (including consumer sentiment, net worth, and personal disposable income, which all

* Again, a more thorough look at this sector can be found in the technical appendix.
go into the consumption equations) and solved the model. This produces data that hopefully will
exactly replicate historical data, although for forecasting purposes an exact replication is not
necessary and is not always the key to a good forecast.

Again, looking at the graph of real simulated GDP (GDPH), and the baseline for real
GDP (GDPH baseline), it can be seen that the model does a fairly good tracking job. The large
gap at the end is mainly attributed to the equations' tracking ability of durable and non-durable
consumption, and essentially shows the errors and residuals associated with these sectors’
modeling ability. Services has a remarkable tracking ability as can be seen from graph 4—this is
an easier part of consumption to model. The equations for all three have a good fit—fairly high
R²'s, low probabilities, and t-tests on the Durbin Watson statistics came out fine. However, it
should also be apparent that durable and non-durable consumption were not tracked as well by
the model (see graphs 2 and 3), thus affecting the model’s ability to track real GDP. This is
something that was mentioned earlier—although all of the pieces are there to track these
variables well, the overall fit of the model is not as good as it could be. Model builders have to
deal with this often; it can be fixed however, using good judgment and is something that comes
with experience.

Section VIII: Discussion of the Model and Structural Equations

As previously mentioned, the CQEM is just under fifty equations and uses over two
hundred variables. This is fairly large for a forecasting model; most models like the larger
Wharton model are used for policy analysis and thus require more precise design and structure.
Actually, a limitation of the model that will surface when examining the multipliers is that it
does not have this structure, and may not trace through some important components of real GDP,
ultimately skewing the multipliers. Despite being able to model the Phillips curve among other important principles, more precision comes with size, and the CQEM cannot compare to models with over eight hundred variables. Forecasting models are used for a limited number of analyses, mainly predictions into the future, and may not be as precisely built to match up to more policy-oriented models. Occasionally forecasters have to go against theory and change something to make the forecast come out better.

The CQEM is composed of seven different sectors: consumption, investment, government, price and income, labor, international and monetary. Each sector has numerous equations in it and when added together, simply should be equal to real GDP (all equations are in real form). There are also many price deflator equations that enable conversion from real to nominal terms, among other identities that help the model define various variables that have been created.*

Real GDP is measured on a chain-weight basis rather than a fixed-weight basis due to rapidly changing prices these days; the data in this model is chain-weighted to 1996 dollars. Thus, a large identity is needed* to convert real GDP back to nominal GDP using chain-weights. A very important part of building a model is how the variables in the model are measured, mainly GDP—are they fixed-weighted or chain-weighted? Both measures of output are quarterly statistics reported by the BEA and can be in terms of any year, usually five years prior. With fixed-weights, the choice of the year in which the weights are based has a large impact on the measurement of real GDP growth and has had a tendency in the past to overstate growth.22 That is, especially in a period of rapidly changing prices, a fixed-weighted measure of GDP can make it hard to compare GDP figures from different years. In 1995, the NIPA were

* See pages ix and x of the technical appendix.

22 Most of this section is based on appendix 14.1 in Pindyck and Rubinfeld (1998), written by Michael Donihue.
rebenchmarking, and the BEA began using a new measure of real GDP called chain-weighted real GDP. This means that instead of measuring real GDP in terms of an arbitrary fixed base year, chain-weighted real GDP is measured by making two calculations of growth for each year using the present year and the preceding year as bases (Steindel 1995). Although this measure does not allow the level of real GDP to be simply equal to the sum of its chain-weighted components \(C + I + G\), converting these in rate of growth terms works. Nominal GDP, as we know, is always in level terms. This concept of using chain-weighted GDP figures is important when talking about the construction of a model, and is a good idea in a time of constantly changing prices.

Since the consumption sector is explained in detail in the technical appendix, the other sectors in the model are worth briefly overviewing to get a sense of the entire model and its components. The last part of the technical appendix is a useful reference in this section of the paper.

The investment sector includes equations for fixed private non-residential investment, fixed private residential investment, private inventories, private non-residential fixed investment, a stock price index (Standard & Poor's 500 Composite), and an industrial production index. The equation for fixed private non-residential investment includes the 5-year treasury note yield variable, private non-residential fixed investment and the industrial production index. The fixed private residential investment equation includes a lagged value of itself, contract rates on 30-year commitments (mortgages), and personal disposable income. The private inventories equation is made up of a lagged value of itself, personal consumption expenditures, the three-month t-bill rate, capacity utilization industry and real GDP. The private non-residential fixed investment
equation includes real GDP and the S&P 500 stock price index; the index itself is made up of corporate profits, the federal funds rate and consumer sentiment. Finally, the industrial production index includes a regressed value of itself and capacity utilization manufacturing. The overall goal of the investment sector is to model productivity levels, capital spending and overall investment in the economy (residential, non-residential, and private).

The government sector, although it only has two equations, is actually endogenizing the government surplus (GFBAL) in the model through a large identity. The first equation is government receipts, which is mainly composed of taxes. Hence, two variables in the equation are personal income and corporate income, as well as a lagged value of government receipts itself. The other government equation is federal government expenditures, which includes a t-bill rate (5-year turned out the best) and a lagged value of itself. These two equations represent a good portion of the government sector, and worked well in forecasting the model.

The price and income sector is made up of many equations. The main goal of this sector, however, is to correctly model the Phillips curve and a general price level, as well as a good estimate of income. The equations in this sector include the rate of growth of gross business product (the Phillips curve, or a chain-weighted price index), a price index including imports of goods and services, the CPI-U (all items less food and energy), the average price of gasoline (all types), the CPI (including food and energy), personal disposable income, transfer payments to persons, personal tax and non-tax payments, personal income, and corporate profits. The CPI and the income equations are straightforward in terms of what they are trying to model and the variables involved; some have been mentioned previously. However, the equation that is trying

*See pg. x of the technical appendix.
to model the Phillips curve is worth noting and talking about. The rate of growth of gross business product is used as a price index in this equation to model the Phillips curve, which shows (usually) the inverse relationship between inflation and unemployment. In recent years, however, there has been some debate about if the Phillips curve still exists due to low levels of inflation and unemployment occurring at the same time. Some have suggested that other factors such as technology have caused the curve to shift, thus causing new standards. In our equation, the rate of growth of gross business product is regressed on itself for nine quarters back to show the rate of growth of inflation. Other variables in the equation used to mimic the curve possibly shifting are the civilian unemployment rate, an annualized form of non-farm business sector output, an annualized form of an import price index, and two variables created to mimic Nixon’s price controls; one variable for when they were not imposed and one for when they were. The modeling of the Phillips curve is an important part of this sector and when studied by itself can lead to some interesting conclusions.

The labor sector includes equations for the civilian unemployment rate, the civilian employment rate (ages 16+), non-farm business sector output, and an index for help-wanted ads in the newspapers. The equation for the civilian unemployment rate uses Okun’s Law in its derivation. Using a generated variable entitled outputgap (potential GDP – actual GDP), civilian unemployment is modeled by regressing this variable three periods back (three lags). The equation for civilian employment is equal to a function of itself regressed one quarter, real GDP, the index of help-wanted ads in newspapers, and non-farm business sector output. The equation for non-farm business sector output is derived as a function of itself regressed one quarter, real GDP, manufacturing capacity utilization, industrial production in the manufacturing sector
regressed one quarter, and industrial production in the manufacturing sector regressed four quarters. Industrial production in the manufacturing sector is regressed twice so that this theory proves true. The equation for the index of help-wanted ads in newspapers is a function of the unemployment rate and industrial capacity utilization.

The international sector consists of only four equations, but includes a very important variable that was generated to mimic GDP for the rest of the world. This variable was created using the U.S.'s four biggest trading partners: Canada, Mexico, the UK, and Japan, and finding the weights of these countries in terms of trading levels. These weights were found to be 5/12, 1/4, 1/6, and 1/6. Exchange rates for these countries were then used to calculate GDP in terms of U.S. dollars. The GDPs were then weighted to form a rest-of-world GDP. Other equations in this sector are the equation for the nominal trade-weighted exchange value of U.S. dollars versus major currencies, exports of goods and services, imports of goods and services, and non-petroleum goods imports. The equation for the nominal trade-weighted exchange value of U.S. dollars versus other major currencies includes a variable similar to the rest-of-world GDP, but was created to mimic a rest-of-world interest rate. This equation also includes nominal imports, the three-month t-bill rate, price deflators, and nominal exports. The export equation includes the rest-of-world GDP variable, along with the exchange rate variable stated above, and a one quarter lag. The import equation is made up of the exchange rate variable, non-petroleum imports, and personal disposable income. And finally, the non-petroleum imports equation includes a lagged variable of itself, imports, and the generated variable used for the rest-of-world interest rate. This sector is based on sound economic theory deriving from the three most important theories of exchange rate determination and international finance: purchasing power
parity (exchange rates between two countries' currencies equals the ratios of the countries price levels), interest rate parity (deposits of all currencies should offer the same expected rate of return), and the J-curve (the time lag in which a real currency depreciation improves a country's current account).

The monetary sector includes equations for the money stock, the three-month t-bill (a money demand equation), the thirty-year mortgage rate, the five- and ten-year treasury note, the federal funds rate (Greenspan's reaction function), and finally Moody's seasonally adjusted AAA corporate bond yield. As one can see, this is a big sector, and is a very important one in modeling the macroeconomy due to all of the interest rate variables. Many of the interest rate equations are solely made up of a lagged value of the variable and another interest rate. This is due to the fact that all interest rates are linked in some way or another. Therefore, the three-month t-bill and the federal funds rate equations are the only ones worth noting because they are unique to this sector and they are also equations for money demand and the Fed's reaction function.

Since money demand is inversely related to interest rates, the theory goes that we can use our three-month t-bill equation to replicate money demand. The equation includes a lagged variable of the three-month t-bill itself, money stock, the rate of growth of gross business product (a measure of inflation/price index), and disposable income. The federal funds rate is composed of last period's federal funds rate, discount window borrowing, and two generated terms: one is a measure of annualized inflation, and the other is a proxy for the output gap. The composition of these two equations is important and many of these variables come into play later when examining the model in more detail.
It should now be more clear how complex these models can get, and how many different equations and variables make up a larger sum that is used to replicate the macroeconomy as a whole. As mentioned before, it is the very nature of these equations and the structure of this model that allows it to be used for certain purposes and not for others. The next few sections will examine in more detail the limitations of this model using a few of the economic repercussions from the events of September 11th to shock the model and examine its dynamic properties though multipliers and other analyses.

Section IX: Discussion of the Shocks

Six months later we have a lot of insight as to how the events of September 11th have affected our lives thus far and how that tragic day will continue to reshape the world as we know it. While we have been experiencing a decline in most forms of economic activity since September 11th, things may be looking up as talked about earlier in this paper.

Using a few of these economic repercussions that came about as a result of the September 11th attacks, the next step in this study is to shock the model, re-simulate it, and examine its dynamic properties through multiplier calculations. The model will then be compared with other models using shocks that have been used in other studies.

After September 11th, the Federal Reserve and the government decided to add a lot of liquidity and funding to the market as to prevent as much shock as it could. Initial estimates of the fiscal stimulus were $70 billion in government spending. The Fed injected a lot of liquidity into the market in the form of reserves to retain confidence, but did not expand the money supply by doing this. In this study, however, this is treated as a permanent increase in the money supply of 2.7% to make the shock easier to perform. Consumer sentiment also took a huge hit in
September, dropping almost a full ten points. These effects are now used to shock the CQEM and examine the outcome.

Before continuing with the discussion of the shocks, more must be explained about how the model was set up to perform this type of examination. Since the model had already been compiled and simulated for forecasting purposes, this was not repeated. However, in order to shock and re-simulate the model, baselines equal to historical tracking simulations had to be created. This means that for each equation and identity, EViews had to be told to add on residuals to make the model replicate historical data exactly. The model is then re-simulated with this new adjustment, and solved from the beginning of 1982 (quarter one). By doing this, one is able to examine the model without worrying if the results are due to the time period the model is shocked in or in fact due to the nature of the model (the latter is what we are trying to examine). This is a very important step in the process of examining a model.

The shock to government spending of $70 billion is the easiest to perform. To replicate this in the model, the data that EViews uses has to be edited. The data for federal consumption expenditures/gross investment (GF), are changed in 1982:1 from $300.6 billion to $370.6 billion. The model is then re-simulated with this new number. Real and nominal GDP figures will change as a result of this shock; the exact change is documented in the form of multipliers, the topic of the next section.

The same is done with consumer sentiment (CSENT), however a different method is used for adjusting the data since it is in index form. The data are edited for this figure by increasing the add-factor residual in 1982:1 from -0.234 to -10 to account for the ten-point drop. The model is then re-simulated with this new shock.

*Note: All shocks are separate from one another—each is performed with the same “clean” historically simulated model (except solving the model with the government spending shock with and without the federal funds rate).
For the monetary shock, the money supply/stock (M2) is increased by 2.7% in 1982:1 by fetching in another set of data for M2, performing this shock, and renaming the old variable so that EViews would use the data set with the new number in it. The model is re-simulated again with M2 exogenized so as to force the new data to be used (that includes the increase in M2).

To examine the Fed influence in the economy, the federal funds rate (the Fed's reaction function) is exogenized when performing the government spending shock and the model is re-simulated to see if the Fed's behavior has much influence. The results, as talked about in the next section, are surprising.

Section X: Discussion of Multipliers and Other Examination Tools

As noted earlier, multipliers are an important part of examining an economy. They are also, however, an important part of examining a model. Depending on the model and how it is compiled, a shock of the same magnitude can produce different multipliers. That is, a shock to government spending of $70 billion can change output by more in one model than in another. In the next section, the CQEM is compared with other models; as one would expect, shocks to the different models do in fact yield different multipliers.

The following table lists the multipliers from the various shocks performed as explained previously. All multipliers are calculated by dividing the change in output (GDP and GDPH) by the amount of the shock to the autonomous or exogenous variable.
Table 1

<table>
<thead>
<tr>
<th>Multipliers-CQEM</th>
<th>Impact Multiplier</th>
<th>Long Run Multiplier (40 Quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Government Spending (GF)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+$70 Billion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.190628</td>
<td>0.208</td>
</tr>
<tr>
<td>GDPH</td>
<td>1.884957</td>
<td>-0.156</td>
</tr>
<tr>
<td><strong>Government Spending (GF)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+70 Billion-Holding Federal Funds Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant (Monetary Policy Held to Base Path)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.190614</td>
<td>0.205</td>
</tr>
<tr>
<td>GDPH</td>
<td>1.884942</td>
<td>-0.157</td>
</tr>
<tr>
<td><strong>Consumer Sentiment (CSENT)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10 points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1.086</td>
<td>0.106</td>
</tr>
<tr>
<td>GDPH</td>
<td>1.727</td>
<td>-0.309</td>
</tr>
<tr>
<td><strong>Money Supply/Stock (FM2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.0000438</td>
<td>-0.00011</td>
</tr>
<tr>
<td>GDPH</td>
<td>0.0000451</td>
<td>0.000589</td>
</tr>
</tbody>
</table>

The multipliers can be interpreted as the following:

With, for example, government spending: a $70 billion increase in government spending leads to an increase of ($70 billion) * (1.19) = $83.3 billion increase in GDP.

The government spending multiplier is the greatest, and surprisingly having the federal funds rate exogenized from the model when solving it with this same shock does not change these multipliers by very much. All of this again, has to do with the nature of the model. In the CQEM, government expenditures (GF) goes directly into GDP initially, however real GDP does not go very far in the model after that. If the second quarter multiplier were listed, it would be very small—close to the long run multiplier. The initial impact multiplier is very large but dampens out quickly because of this. Real GDP does go directly into the price and income sector, the labor sector, and the investment sector. However, the feedback effect is only felt through disposable income (in the price and income sector), which feeds directly into
consumption, and investment (which leads back to real GDP). The model also does not include a government employment sector which usually has a large feedback effect component. These are limitations of the model and show problems with its theoretical properties. The multipliers not changing when the Fed funds reaction function is exogenized (i.e. there is no monetary policy reaction—in this model, no federal funds rate) is also a limitation of the model and proves unrealistic. The problem here is that the federal funds rate (FFED) is not a variable in the three-month t-bill rate equation, and thus has no effect on the term structure of interest rates. The federal funds rate also goes into the money supply/stock variable (FM2) as a change variable which dampens the effect of its significance in the model. From the long run multipliers in either case, we can see how the effects of a one-time increase in government spending, while has a large initial effect, dampens out significantly as time goes on.

Consumer sentiment, however, is a different story. Although the multiplier is lower, the feedback loop for consumer sentiment in the model is defined better—consumer sentiment goes directly into consumption in two important equations which directly lead back to real GDP. In this way, it is more effective than government spending which can be seen due to the fact that the multipliers after the impact multiplier dampen out, but do not go directly to zero (or close to it) the next quarter as the government spending multipliers did.

Since the money supply’s feedback effect is very roundabout in the model, it is no surprise that the change in FM2 does not bring a large multiplier. Again, this multiplier is rather damp because of the structure of the model. Through the three-month t-bill rate, FM2 goes into inventories (investment sector), and the international sector (imports and exports), which both lead back to real GDP—but it is a remote connection.
The effects of the shocks can also be seen by calculating the percent-change in real GDP and nominal GDP. Below are sets of graphs depicting these percent changes:

The percent change in GDP due to the increase in government spending is around 2.7%—a fairly significant increase. The huge drop off in the percent change of GDP in the second quarter can be seen (it becomes negative, but note the scale). As with the second quarter multiplier mentioned above, this drop off is due to the construction of the model and the fact that GDP does not flow back into the model very well once the initial impact of the increase in government spending has taken its toll. After the first two quarters, it looks as though things get back to normal, with the percent change in GDP varying very little and staying close to zero.

* These percent changes show the deviations of GDP and real GDP from the baseline, calculated:

\[
\frac{((\text{GDP with shock})-\text{baseline GDP})}{\text{baseline GDP}}
\]
In the following graph, it can be noted that when the Fed's reaction function is exogenized, GDP does not change by very much. That is, as mentioned before, if the Fed did nothing this entire time in terms of monetary policy, nominal GDP would be the same as it was with the increase in spending including the Fed's reaction function.

Intuitively, this does not make a lot of sense—having the Fed involved with the economy is the same as when the Fed does nothing. However, again, this is due to the structure of the model and not having the federal funds rate in the three-month t-bill equation thus eliminating the term structure of interest rate effect on the economy. This is a huge effect because it links the federal funds rate with the term structure of other interest rates, such as the three month t-bill. That is, when the federal funds rate changes, due to actions of the Fed previously mentioned, the three-month t-bill rate will change with it. This is because if maturities on these two "loans" were equal (although federal funds loans generally have a shorter maturity), the risk on federal
funds loans is greater and so there is a risk premium that differentiates the two rates. This is due to the fact that the government always has taxation as an option for paying off loans and banks do not. If the federal funds rate changes, the t-bill rate will as well because the risk premium between a treasury security and a federal funds loan will be the same if the change in the federal funds rate is viewed as permanent. These rate changes will, in turn, change the opportunity cost of holding money versus bonds or other securities. That is, when the Fed lowers interest rates, the way this affects the economy is through this link. Without it, the Fed has no effect on the economy, as seen in the multipliers and percent changes above.

The graph above shows us that with a ten-point decrease in consumer sentiment, the percent change in real and nominal GDP is negative at first (-0.35), which makes sense, and gradually becomes less negative in the next ten quarters. There is an obvious lag effect, as the percent decrease is initially smaller than it is two quarters later. This is because consumer sentiment does not feed directly into real and nominal GDP as government spending does. It
feeds directly into the consumption sector which in turn effects real GDP directly. This effect may take a few quarters to pan out as the results show. The difference in real and nominal GDP can be attributed to the fact that consumer sentiment is fed directly into real GDP through consumption, and from there fed into nominal GDP through price deflator calculations (a more remote connection). From the scale, it can be noted that there is not a very large effect on GDP or GDPH as a result of this shock, but this is because the shock is not very large. Consumer sentiment has fallen by more than ten points before, which as can be seen, would have a fairly significant effect on the economy.

The graph above shows that nominal GDP and real GDP both increase with a 2.7% increase in the money supply (stock). A lag is apparent, and is ultimately felt because M2 feeds into the three-month t-bill rate which, through investment, feeds into real GDP. This is the reason for the large jump in the percent change five quarters later. The difference in GDP and
GDPH after five quarters can be attributed to inflation: the Phillips curve, which is modeled in the price and income sector, has nine lags in it, and once this kick in it is easy to note the difference between real and nominal GDP which is now reflective of inflation. This is consistent with theory that an increase in the money supply leads to inflation.

How do these results compare with other models? This is the topic of the next section.

Section XI: Comparison of the CQEM to Other Models

Many macroeconometric models exist and are used by a wide variety of economists for different purposes. The United States is the epicenter for empirical work in macroeconometric model building and has many in use as of today.23 There are many seminars and books dedicated to model comparison, and much enthusiasm is felt among economists to share their new ideas.

While Jan Tinbergen developed the first U.S. macro model with the intent of testing economic theories of the business cycle, many others have moved beyond that into policy analysis and to even incorporating expectations into models. Many things were also impossible to examine without the computer such as forecasting, multiplier analysis, and stochastic simulation, which can now be done with a fast turnaround. The creation of the vector autoregressive systems (VAR—by the Federal Reserve Bank of Minneapolis and the University of Minnesota) is a revolution in policy analysis in that now when specific policy changes are introduced in a model, not just the exogenous variables are shocked, but the equations are changed as well. This is a direct result of the work of Robert Lucas and his critique of using models for policy analysis. The introduction of ARIMA models along with VAR models now allows for more precise forecasting in the short run (present in the CQEM—(AR1 process)), and helps with adjustment processes.

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23 This section is referenced from Klein (1991).
In his book entitled *Comparative Performance of U.S. Econometric Models* (1991), Lawrence R. Klein, along with F. Gerard Adams, examines the performance of ten econometric models by doing the same thing this study does—shocking each model and calculating average dynamic multipliers for each shock. A few of the models included are that of the Bureau of Economic Analysis, Ray Fair, and the University of Michigan. The models range in size (most have several hundred equations), composition, and purpose, and all start from a tracking solution to reproduce history exactly over the period 1975:1 to 1984:4.

One shock that could be reproduced given the make up of the CQEM is what the authors call the "spending shock". In this shock, government spending is shocked by 1% of historical GDP for forty quarters with a monetary instrument held to base path (the federal funds rate in the CQEM). That is, GDP is increased by 1% for forty quarters, the models are simulated again, and multipliers are calculated in the same way as they are earlier in this paper. The resulting multipliers of the shocks are in the table below:

<table>
<thead>
<tr>
<th>Quarter</th>
<th>CQEM</th>
<th>Adams and Klein's Average Model Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.19</td>
<td>1.06</td>
</tr>
<tr>
<td>2</td>
<td>1.11</td>
<td>1.42</td>
</tr>
<tr>
<td>3</td>
<td>1.21</td>
<td>1.54</td>
</tr>
<tr>
<td>4</td>
<td>1.28</td>
<td>1.55</td>
</tr>
<tr>
<td>40</td>
<td>4.63</td>
<td>0.15</td>
</tr>
</tbody>
</table>

It is easy to note a difference in the two sets of multipliers—an interesting result. Whereas the CQEM's multipliers for this spending shock constantly increase, the average model study multipliers done by Adams and Klein decrease. This has to do, again, with the make up of the model and its theoretical properties. The CQEM has accelerated properties due to the lag
structure which skews the multipliers. That is, for most equations there is a lag present as one of the variables. This means that every quarter is a function of last periods value (say, for government spending) plus other variables. This would always make the multipliers larger as time goes on—the values of the variables are constantly building on previous values. The different uses of the models can cause a large difference in how they will respond to various shocks; the average models in the study are mainly used for policy analysis purposes and the CQEM was built for forecasting purposes. Whereas the average models should be able to replicate history almost exactly so they can be used to decide the impact of a certain policy action, forecasting models only need to produce good forecasts into the future. Forecasting models could be very far off in accuracy of reproducing history, while predicting the future with great preciseness.

While this is an interesting comparison, comparing models, especially through multipliers, is a difficult task. The average models in Klein and Adams' study are undoubtedly larger and thus have more components to model the economy as a whole better. Most include expectations in some form or another (surveys for investment expectations, housing starts, consumer sentiment), and all the participants agreed on certain inputs in the models so that they could be compared (this took more than a year to decide). Without this requirement, there are problems with units of measurement, price deflator calculations, dynamic paths and model adjustments. All of these problems with comparing models are evident when examining the dynamic multipliers, though the results are interesting to note.
Section XII: Concluding Remarks

While the CQEM may not match up to other larger and well renowned models, it does a good job modeling the economy for forecasting purposes and overall is well put-together. This model was not meant for policy analyses and hence was not built to be able to examine this. Its theoretical properties are the way they are because the model produced the best forecast when built this certain way. In this regard, the Senior Forecasting Seminar of Colby College did a good job.

The new models such as the ones in the study by Klein and Adams have progressed immensely since the days of Robert Lucas. While there are still criticisms of models today and how accurate they are in replicating our economy, it is reassuring that these criticisms are being thought about and problems are being solved. Models are becoming increasingly used by economists today especially with new computer revelations in this field, and will probably keep becoming more and more complex with time. Comparing models is also becoming increasingly more important as a way to improve existing models and create new ways to model the different sectors better. Seminars and lectures held are in this way becoming an integral part of the maturing of the macroeconometric model, among others.

The events of September 11th, while tragic, will ultimately lead to some interesting research in economics. Never before have economists seen every sector of the economy react to an event of such magnitude at the same time. A lot of research needs to be done in this era of our "new economy" that has formed in the last decade or so. Only then will models like the Colby Quarterly Econometric Model be able to thoroughly explain the economic effects of a day like September 11th.
Technical Appendix
Abbreviations of Variables
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>Personal Consumption Expenditures: Durable Goods (SAAR, Bil.$)</td>
</tr>
<tr>
<td>CDH_A</td>
<td>Personal Consumption Expenditures: Services (SAAR, Bil.$)</td>
</tr>
<tr>
<td>CSENT</td>
<td>University of Michigan: Consumer Sentiment (66Q1=100, NSA)</td>
</tr>
<tr>
<td>CH</td>
<td>Capacity Utilization: Manufacturing (SA, Percent of Capacity)</td>
</tr>
<tr>
<td>CN</td>
<td>Capacity Utilization: Industry (SA, Percent of Capacity)</td>
</tr>
<tr>
<td>CS</td>
<td>Nominal Trade-Weighted Exch Value of US$ vs Major Currencies (MAR 73=1)</td>
</tr>
<tr>
<td>CSENT_A</td>
<td>Federal Government Current Receipts (SAAR, Bil.$)</td>
</tr>
<tr>
<td>FCM</td>
<td>Contract Rates on Commitments: Conventional 30-Yr Mortgages, FHLMC (%)</td>
</tr>
<tr>
<td>FCM10</td>
<td>10-Year Treasury Bond Yield at Constant Maturity (%)</td>
</tr>
<tr>
<td>FCM10_A</td>
<td>3-Month Treasury Bills, Secondary Market (%)</td>
</tr>
<tr>
<td>FCM5_A</td>
<td>Money Stock: M2 (SA, Bil.$)</td>
</tr>
<tr>
<td>FCMS</td>
<td>Government Consumption Expenditures/Gross Investment (SAAR, Bil.$)</td>
</tr>
<tr>
<td>FCM</td>
<td>Gross Domestic Product (SAAR, Bil.$)</td>
</tr>
<tr>
<td>FCMH</td>
<td>5-Year Treasury Note Yield at Constant Maturity (%)</td>
</tr>
<tr>
<td>FCMB</td>
<td>Federal Government Surplus or Deficit [-], NIPA (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GC</td>
<td>Federal Government Current Receipts (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GC_A</td>
<td>Federal Government Wage Accruals less Disbursements (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GFSUBX</td>
<td>Federal Subsidies less Current Surplus of Govt Enterprises (SAAR,Bil.$)</td>
</tr>
<tr>
<td>GFTF</td>
<td>Government Consumption Expenditures/Gross Investment (SAAR,Bil.Chn.96$)</td>
</tr>
<tr>
<td>G</td>
<td>Government Consumption Expenditures/Gross Investment (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GDPH</td>
<td>Gross Domestic Product (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GDPM</td>
<td>Gross Domestic Product (SAAR, Bil.Chn.1996$)</td>
</tr>
<tr>
<td>GDPPOTHQ</td>
<td>Nominal Trade-Weighted Exch Value of US$ vs Major Currencies (MAR 73=1)</td>
</tr>
<tr>
<td>GDPROW</td>
<td>Rest of World GDP</td>
</tr>
<tr>
<td>GF</td>
<td>Federal Consumption Expenditures/Gross Investment (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GFSUBX</td>
<td>Federal Subsidies less Current Surplus of Govt Enterprises (SAAR,Bil.$)</td>
</tr>
<tr>
<td>GFTF</td>
<td>Federal Government Transfer Payment (Net) (SAAR, Bil.$)</td>
</tr>
<tr>
<td>GH</td>
<td>Government Consumption Expenditures/Gross Investment (SAAR,Bil.Chn.96$)</td>
</tr>
<tr>
<td>GS</td>
<td>State &amp; Local Consumption Expenditures &amp; Gross Investment (SAAR,Bil.$)</td>
</tr>
<tr>
<td>IP</td>
<td>Industrial Production Index (SA, 1992=100)</td>
</tr>
<tr>
<td>IP_A</td>
<td>Industrial Production Index (SA, 1992=100)</td>
</tr>
<tr>
<td>IPMFG</td>
<td>Industrial Production: Manufacturing (SA, 1992=100)</td>
</tr>
<tr>
<td>JGDPB</td>
<td>Gross Business Product: Chain Price Index (SA, 1996=100)</td>
</tr>
<tr>
<td>Variable</td>
<td>Equation</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>JM</td>
<td>Eq44</td>
</tr>
<tr>
<td>JMMX</td>
<td>Eq60</td>
</tr>
<tr>
<td>LE</td>
<td>Eq55</td>
</tr>
<tr>
<td>LF</td>
<td>Eq17</td>
</tr>
<tr>
<td>LHELP</td>
<td>Eq54</td>
</tr>
<tr>
<td>LR</td>
<td>Eq53</td>
</tr>
<tr>
<td>LR_A</td>
<td>Eq53</td>
</tr>
<tr>
<td>LXNFA</td>
<td>Eq56</td>
</tr>
<tr>
<td>M</td>
<td>Eq12</td>
</tr>
<tr>
<td>MH_A</td>
<td>Eq58</td>
</tr>
<tr>
<td>NETWORTH</td>
<td>Eq34</td>
</tr>
<tr>
<td>NIXOFF</td>
<td>Exog</td>
</tr>
<tr>
<td>NIXON</td>
<td>Exog</td>
</tr>
<tr>
<td>OUTPUTGAP</td>
<td>Eq18</td>
</tr>
<tr>
<td>PCU</td>
<td>Eq47</td>
</tr>
<tr>
<td>PCUSLFE</td>
<td>Eq45</td>
</tr>
<tr>
<td>PDC</td>
<td>Eq19</td>
</tr>
<tr>
<td>PDCD</td>
<td>Eq21</td>
</tr>
<tr>
<td>PDCN</td>
<td>Eq22</td>
</tr>
<tr>
<td>PDCS</td>
<td>Eq23</td>
</tr>
<tr>
<td>PDFN</td>
<td>Eq24</td>
</tr>
<tr>
<td>PDFR</td>
<td>Eq25</td>
</tr>
<tr>
<td>PDG</td>
<td>Eq26</td>
</tr>
<tr>
<td>PDG_A</td>
<td>Eq26</td>
</tr>
<tr>
<td>PDGDP</td>
<td>Eq20</td>
</tr>
<tr>
<td>PDM</td>
<td>Eq27</td>
</tr>
<tr>
<td>PDS</td>
<td>Eq28</td>
</tr>
<tr>
<td>PDX</td>
<td>Eq29</td>
</tr>
<tr>
<td>PGAS</td>
<td>Eq46</td>
</tr>
<tr>
<td>PMOIL</td>
<td>Exog</td>
</tr>
<tr>
<td>ROG.GDPB</td>
<td>Eq43</td>
</tr>
<tr>
<td>ROWR</td>
<td>Exog</td>
</tr>
<tr>
<td>S</td>
<td>Eq10</td>
</tr>
<tr>
<td>SH</td>
<td>Eq37</td>
</tr>
<tr>
<td>SH_A</td>
<td>Eq37</td>
</tr>
<tr>
<td>SP500</td>
<td>Eq39</td>
</tr>
<tr>
<td>X</td>
<td>Eq11</td>
</tr>
<tr>
<td>XH_A</td>
<td>Eq57</td>
</tr>
<tr>
<td>YCP</td>
<td>Eq52</td>
</tr>
<tr>
<td>YP</td>
<td>Eq51</td>
</tr>
<tr>
<td>YPDH</td>
<td>Eq48</td>
</tr>
<tr>
<td>YPTP</td>
<td>Eq49</td>
</tr>
<tr>
<td>YPX</td>
<td>Eq50</td>
</tr>
</tbody>
</table>
The Consumption Sector

Flowchart:

\[ \log(CD)_t = -3.22 + 0.64\log(CD)_{t-1} + 0.53\log(YPDH)_{t-1} + 0.20\log(CSENT)_{t-1} - 0.02\log(FCMS)_{t} + 0.01(d\log(JGDPB)*400)_{t} \]

T-statistic: (-8.50) (14.51) (7.87) (7.65) (-2.65) (3.52)
Probability: (0.00) (0.00) (0.00) (0.00) (0.0087) (0.006)

\[ R^2 = 0.998 \quad DW = 2.21 \]

\[ \log(CNH) = 0.58 + 0.54\log(CNH)_{t-1} + 0.27\log(YPDH)_{t-1} + 0.09\log(NETWORTH)_{t-1} - 0.12\log(PDCN)_{t} + 0.61AR(1) \]

T-Statistic: (5.12) (8.79) (6.64) (6.86) (-7.02) (7.08)
Probability: (0.00) (0.00) (0.00) (0.00) (0.00) (0.00)

\[ R^2 = 0.999 \quad DW = 2.37 \]

\[ CSH_t = -6.65 + 0.86CSH_{t-1} + 0.07\log(YPDH)_{t-1} + 0.01\log(NETWORTH)_{t-1} + 28.8d\log(CSENT)_{t} - 0.90\log(FCMS)_{t} + 0.16AR(1) \]

T-Statistic: (-1.50) (36.06) (5.77) (2.74) (3.53) (-2.53) (2.09)
Probability: (0.134) (0.00) (0.00) (0.006) (0.0005) (0.012) (0.03)

\[ R^2 = 0.999 \quad DW = 2.00 \]

\[ d\log(PDCD)_{t} = -0.002 + 0.85d\log(JGDPB)_{t} + 0.50AR(1) \]

T-Statistic: (-3.08) (10.07) (7.05)
Probability: (0.0024) (0.00) (0.00)

\[ R^2 = 0.75 \quad DW = 1.97 \]

\[ d\log(PDCN)_{t} = -0.001 + 1.14d\log(JGDPB)_{t} + 0.32AR(1) \]

T-Statistic: (-1.71) (15.94) (4.88)
Probability: (0.0891) (0.00) (0.00)

\[ R^2 = 0.70 \quad DW = 2.04 \]

\[ PCDS_t = -41.87 + 0.97PCDS_{t-1} + 0.74\log(JGDPB)_{t} + 0.74AR(1) \]

T-Statistic: (-5.21) (4.55) (5.51) (8.53)
Probability: (0.00) (0.001) (0.00) (0.00)

\[ R^2 = 0.99 \quad DW = 2.38 \]

---

1 Note: the identities for consumption as a whole (CX) and real consumption (CH) can be found above on the next page under "Nominal and Real Consumption." Also, to keep things simple, only the consumption equations are listed above along with the price deflators that convert real values to nominal values. Variables such as CSENT, YPDH, NETWORTH, and JGDPB do have corresponding equations not shown here.
where: CDR = Real durable consumption
CNH = Real non-durable consumption
CSH = Real services consumption
YPDH = Real disposable income
CSENT = Consumer sentiment (index)
FCM5 = 5-year Treasury Note
JGDPB = Gross Domestic Business Product
NETWORTH = Net Worth
PDCD = Price Deflator for durable consumption
PDCN = Price Deflator for non-durable consumption
PDCS = Price Deflator for services consumption

The Theory Behind the Consumption Sector Equations

CDH Equation (durable consumption): Mostly log format—rate of growth of cdh is what matters

• Regressed on one lagged value of itself to account for past durable consumption.
• Consumption of durable goods is dependent on personal disposable income (YPDH); if one has more income, they will buy more of these bigger-ticket items.
• CDH is also dependent on how consumers are feeling at the time (consumer sentiment-CSENT); if they feel good about the economy and their financial position, they will consume more.
• Nominal interest rates matter for durable consumption because as interest rates fall, consumers will consume more because the opportunity cost of holding money goes down with interest rates. The five-year t-note rate (FCM5) is used because this is the most relevant interest rate for consumers.
• The dlog function was used on the annual rate of inflation variable (JGDPB) because the percent rate of growth is what matters to measure this.

CNH Equation (non-durable consumption): All logs—rate of growth of cnh is what matters

• Regressed on one lagged value of itself to account for past non-durable consumption.
• Consumption of non-durable goods is a function of personal disposable income; if people have more income, then they will usually consume more necessary items, although CNH is fairly income inelastic.
• CNH is a function of net worth as well, for the same reason as income, although net worth also includes assets one owns to add to their total value, not just income and wealth.

• There is an AR(1) process (autoregressive) added to take care of a serial correlation problem that existed.

CSH Equation (services consumption):

• Regressed on itself one lag to account for past values.

• Personal disposable income is a function of how many services one consumes, the higher their income, the more services they will want.

• Net worth is a factor for the same reason as income although it includes assets one owns as well as their wealth.

• The percent rate of growth is what matters for consumer sentiment; the better people feel, the more services they consume.

• The nominal 5-month t-note is important for consumers; the higher the nominal interest rate on these, the higher the opportunity cost of consuming services. Services are more nominally interest rate sensitive because they can be completely substituted with the securities market. No one has to travel or go see a play if they think the opportunity cost of not having that money in say, bonds, is high enough.

• There is an AR(1) process to take care of a serial correlation problem that existed.

The price deflators for each sub-sector are mainly regressed on the rate of growth of gross business product (for PDCD and PDCN) and just gross domestic business product (for PDCS) so that all of the above real equations can be converted to nominal form.
Graph 1: Real GDP (Baseline and Simulated)

Graph 2: Real Durable Consumption (Baseline and Simulated)
IDENTITIES:

Nominal GDP
\[ \text{gdp} = \text{cd} + \text{cn} + \text{cs} + \text{fn} + \text{fr} + \text{s} - \text{s}(-1) + \text{g} + \text{x} - \text{m} \]

Real GDP - Chain weighting means that the NIPA identity doesn't work in real terms. This is an approximation.

\[ \text{gdph} = \text{gdph}(-1) \times ((\text{cd}(-1) / \text{gdp}(-1)) \times (\text{cdh} / \text{cdh}(-1))) + ((\text{cn}(-1) / \text{gdp}(-1)) \times (\text{cnh} / \text{cnh}(-1))) + ((\text{cs}(-1) / \text{gdp}(-1)) \times (\text{csh} / \text{csh}(-1))) + ((\text{fn}(-1) / \text{gdp}(-1)) \times (\text{fnh} / \text{fnh}(-1))) + ((\text{fr}(-1) / \text{gdp}(-1)) \times (\text{frh} / \text{frh}(-1))) + (\text{s}(-1) / \text{gdp}(-1)) \times (\text{sh} / \text{sh}(-1))) - ((\text{s}(-2) / \text{gdp}(-1)) \times (\text{sh}(-1) / \text{sh}(-2))) + ((\text{g}(-1) / \text{gdp}(-1)) \times (\text{gh} / \text{gh}(-1))) + ((\text{x}(-1) / \text{gdp}(-1)) \times (\text{xh} / \text{xh}(-1))) - ((\text{m}(-1) / \text{gdp}(-1)) \times (\text{mh} / \text{mh}(-1)))) \]

Nominal & Real Consumption
\[ \text{cx} = \text{cd} + \text{cn} + \text{cs} \]
\[ \text{ch} = \text{ch}(-1) \times ((\text{cd}(-1) / \text{cx}(-1)) \times (\text{cdh} / \text{cdh}(-1))) + ((\text{cn}(-1) / \text{cx}(-1)) \times (\text{cnh} / \text{cnh}(-1))) + ((\text{cs}(-1) / \text{cx}(-1)) \times (\text{csh} / \text{csh}(-1))) \]

Nominal series from deflators
\[ \text{cd} = \text{pdc}d / 100 \times \text{cdh} \]
\[ \text{cn} = \text{pdcn} / 100 \times \text{cnh} \]
\[ \text{cs} = \text{pdc}c / 100 \times \text{csh} \]
\[ \text{fn} = \text{pdf}n / 100 \times \text{fnh} \]
\[ \text{fr} = \text{pdf}r / 100 \times \text{frh} \]
\[ \text{s} = \text{pds} / 100 \times \text{sh} \]
\[ \text{x} = \text{pdx} / 100 \times \text{xh} \]
\[ \text{m} = \text{pdm} / 100 \times \text{mh} \]

Nominal & Real Government Spending
\[ \text{gf}d\text{f} \text{is exogenous.} \]
\[ \text{gf}n\text{h} \text{is exogenous.} \]
\[ \text{gs} \text{is exogenous.} \]
\[ \text{g} = \text{gf} + \text{gs} \]
\[ \text{gh} = \text{g} / (\text{pdg} / 100) \]
Federal Budget Surplus identity:  
: eqgfei is endogenous - see the Government Sector.  
: eqgr is endogenous - see the Government Sector.  
gff is exogenous.  
gfeg is exogenous.  
gfew is exogenous.  
gfsubx is exogenous.  
gfe is exogenous.  

\[ g_{\text{bal}} = g_{\text{fr}} - (g_{\text{fe}} + g_{\text{ftf}} + g_{\text{feg}} + g_{\text{fsubx}} + g_{\text{few}} + g_{\text{fei}}) \]

Other Identities

\[ j_{\text{gdpb}} = j_{\text{gdpb}(-1)} \times (1 + (\text{rogj}_{\text{gdpb}} / 100)\,^\text{.25} \]

\[ l_{\text{f}} = \frac{l_{\text{e}}}{(1 - (\text{lr} / 100))} \]

\[ \text{outputgap} = ((\text{gdph} / \text{gdph(-1)})^{\text{.4}} - 1) \times 100 - ((\text{gdppothq} / \text{gdppothq(-1)})^{\text{.4}} - 1) \times 100 \]

\[ p_{\text{dc}} = c_{\text{x}} / c_{\text{h}} \times 100 \]

\[ p_{\text{dgd}} = g_{\text{dp}} / g_{\text{dph}} \times 100 \]

Components of All Equations

COLBY QUARTERLY ECONOMETRIC MODEL OF THE US ECONOMY (CQEM)

Version: December 5, 2001

IDENTITIES:  
Nominal GDP

Eq1: \[ g_{\text{dp}} = F(c_{\text{d}}, c_{\text{n}}, c_{\text{s}}, f_{\text{n}}, l_{\text{r}}, g, m, s, x) \]
Real GDP - Chain weighting means that the NIPA identity doesn't work in real terms  
This is an approximation.

Eq2: \[ g_{\text{dph}} = F(c_{\text{d}}, c_{\text{dh}}, c_{\text{n}}, c_{\text{nh}}, c_{\text{s}}, c_{\text{sh}}, f_{\text{n}}, f_{\text{nh}}, f_{\text{r}}, f_{\text{rh}}, g, g_{\text{dp}}, g_{\text{dph}}, g_{\text{h}}, m, m_{\text{h}}, s, s_{\text{h}}, x, x_{\text{h}}) \]

Nominal & Real Consumption

Eq3: \[ c_{\text{x}} = F(c_{\text{d}}, c_{\text{n}}, c_{\text{s}}) \]
Eq4: \[ c_{\text{h}} = F(c_{\text{d}}, c_{\text{dh}}, c_{\text{n}}, c_{\text{nh}}, c_{\text{s}}, c_{\text{sh}}, c_{\text{x}}) \]

Nominal series from deflators

Eq5: \[ c_{\text{d}} = F(c_{\text{dh}}, p_{\text{cd}}) \]
Eq6: \[ c_{\text{n}} = F(c_{\text{nh}}, p_{\text{cn}}) \]
Eq7: \[ c_{\text{s}} = F(c_{\text{sh}}, p_{\text{cs}}) \]
Eq8: \[ f_{\text{n}} = F(f_{\text{nh}}, p_{\text{fn}}) \]
Eq9: \( fr = F(fr_h, pdfr) \)
Eq10: \( s = F(pds, sh) \)
Eq11: \( x = F(pdx, xh) \)
Eq12: \( m = F(m_h, pdm) \)

Nominal & Real Government Spending

gfdi is exogenous.
gfni is exogenous.
gs is exogenous.

Eq13: \( g = F(gf, gs) \)
Eq14: \( gh = F(g, pdg) \)

Federal Budget Surplus identity:

\texttt{eqgfei} is endogenous - see the Government Sector.

\texttt{eqgfr} is endogenous - see the Government Sector.

gfft is exogenous.
gfeg is exogenous.
gfew is exogenous.
gfsubx is exogenous.
gfe is exogenous.

Eq15: \( g_{bal} = F(gfe, gfeg, gfei, gfew, gfr, gfsubx, gfft) \)

Other Identities

Eq16: \( j_{gdpb} = F(j_{gdpb}, rog_{jgdpb}) \)
Eq17: \( l_f = F(l_e, lr) \)
Eq18: \( \text{outputgap} = F(gdph, gdppothq) \)
Eq19: \( pdc = F(ch, cx) \)
Eq20: \( pdgdp = F(gdp, gdph) \)

\textbf{STRUCTURAL EQUATIONS}

Price Deflators

Eq21: \( pdc_d = F(j_{gdpb}, pdc_d) \)
Eq22: \( pdc_n = F(j_{gdpb}, pdc_n) \)
Eq23: \( pdc_s = F(j_{gdpb}, pdc_s) \)
Eq24: \( pdn = F(pdff, rog_{jgdpb}) \)
Eq25: \( pdfr = F(pdfr, rog_{jgdpb}) \)
Eq26: \( pdg = F(pdg, rog_{jgdpb}) \)
Eq27: \( pdm = F(j_{mmx}, pdm) \)
Eq28: \( pds = F(pds, rog_{jgdpb}) \)
Eq29: \( pdx = F(j_{gdpb}, pdx) \)

Consumption Sector

Eq30: \( cdh = F(cdh, csent, fcsm5, j_{gdpb}, ypdh) \)
Eq31: \( cnh = F(cnh, networth, pdcn, ypdh) \)
Eq32: \( csh = F(csent, csh, fcsm5, networth, ypdh) \)
Eq33: \( csent = F(csent, gfbal, lr, rog_{jgdpb}, ypdh) \)

Net worth is an AR(1)

Eq34: \( \text{networth} = F(\text{networth}) \)

Investment Sector

Eq35: \( fnh = F(fcm5, fnenph, ip) \)
Eq36: \( \text{frh} = F(\text{fcm}, \text{frh}, \text{ypdh}) \)
Eq37: \( \text{sh} = F(\text{ch}, \text{cut}, \text{ftbs3}, \text{gdph}, \text{sh}) \)
Eq38: \( \text{fnenph} = F(\text{gdph}, \text{sp500}) \)
Eq39: \( \text{sp500} = F(\text{csemt}, \text{fled}, \text{sp500}, \text{ypc}) \)
Eq40: \( \text{ip} = F(\text{cumfg}, \text{ip}) \)

**Government Sector**

Eq41: \( \text{gfei} = F(\text{fcm5}, \text{gfei}) \)
Eq42: \( \text{gfr} = F(\text{gfr}, \text{ycp}, \text{yp}) \)

**Price & Income Sector**

Eq43: \( \text{rogjgdpb} = F(\text{jm}, \text{lr}, \text{lxnfa}, \text{nixoff}, \text{nixon}, \text{pcusife}, \text{rogjgdpb}) \)
Eq44: \( \text{jm} = F(\text{jm}, \text{m}) \)
Eq45: \( \text{pcusife} = F(\text{pcusife}, \text{rogjgdpb}) \)
Eq46: \( \text{pgas} = F(\text{cnh}, \text{pgas}, \text{pmoil}) \)
Eq47: \( \text{pcu} = F(\text{pcu}, \text{rogjgdpb}) \)
Eq48: \( \text{ypdh} = F(\text{gdph}, \text{ypdh}, \text{ypdp}, \text{ypx}) \)
Eq49: \( \text{ypdp} = F(\text{gdph}, \text{ypdp}, \text{ypx}) \)
Eq50: \( \text{ypx} = F(\text{yp}, \text{ypx}) \)
Eq51: \( \text{yp} = F(\text{gdph}) \)
Eq52: \( \text{ycp} = F(\text{csemt}, \text{lxnfa}, \text{ycp}, \text{yp}) \)

**Labor Sector**

Eq53: \( \text{lr} = F(\text{lr}, \text{outputgap}) \)
Eq54: \( \text{lhelp} = F(\text{cut}, \text{lhelp}, \text{lr}) \)
Eq55: \( \text{le} = F(\text{gdph}, \text{le}, \text{lhelp}, \text{lxnfa}) \)
Eq56: \( \text{lxnfa} = F(\text{cumfg}, \text{gdph}, \text{ipmfg}, \text{lxnfa}) \)

**International Sector**

Note: in the real exports equation historically,
\[
gdprow = \frac{5}{12} \cdot \text{gdpcan} + \frac{1}{4} \cdot \text{gdpmex} + \frac{1}{6} \cdot \text{gdpuk} + \frac{1}{6} \cdot \text{gdpjap}
\]
but gdprow is exogenous ex ante

Eq57: \( \text{xh} = F(\text{ftxwm}, \text{gdprow}, \text{xh}) \)
Eq58: \( \text{mh} = F(\text{ftxwm}, \text{jmxx}, \text{ypdh}) \)
Eq59: \( \text{ftxwm} = F(\text{ftbs3}, \text{ftxwm}, \text{m}, \text{pdm}, \text{pdx}, \text{rowr}, \text{x}) \)
Eq60: \( \text{jmxx} = F(\text{jmxx}, \text{m}, \text{rowr}) \)

**Monetary Sector**

Eq61: \( \text{fcm} = F(\text{fcm10}) \)
Eq62: \( \text{fcm10} = F(\text{fcm10}, \text{fcm5}) \)
Eq63: \( \text{fcm5} = F(\text{fcm5}, \text{ftbs3}) \)
Eq64: \( \text{ftbs3} = F(\text{fm2}, \text{ftbs3}, \text{rogjgdpb}, \text{ypdh}) \)
Eq65: \( \text{fm2} = F(\text{ffed}, \text{fm2}) \)
Eq66: \( \text{ffed} = F(\text{fdwb}, \text{ffed}, \text{outputgap}, \text{rogjgdpb}) \)
Eq67: \( \text{faaa} = F(\text{faaa}, \text{fcm10}) \)
References


