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Detecting Stock Market Bubbles: A Price-to-Earnings Approach

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Econometric Bubble Detection

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Abstract

To this day, economists argue about the existence of stock market bubbles. The literature review for this paper observes the analysis of four reputable bubble tests in an attempt to provide ample qualitative proof for the existence of bubbles. The first obstacle for creating an effective bubble detection test is the difficulty of estimating true fundamental values for equities. Without adequate estimations for the fundamental values of equities, the deviation between actual price and fundamental price is impossible to observe or estimate. Additionally, these tests are reliant on strong underlying assumptions, which tend to cloud results.

This thesis applies a price-to-earnings ratio test adapted from a thesis written by Bram Weites and Malte von Maravic (2010). The model utilizes a relationship between the risk and price-to-earnings ratios of equities to econometrically test for bubbles. The test has an advantage over previous bubble literature because it does not require the estimation of the fundamental values of equities. A rolling regression is applied to the econometric model, and four bubbles are detected. The Dot-com bubble is detected with complete confidence, and three other bubbles are detected with slightly less confidence.

1 Introduction

A stock market bubble occurs when composite market prices deviate from composite fundamental values. Although the Dot-com bubble serves as extremely solid evidence for bubbles, many economists deny the existence of bubbles in financial markets. For example, the efficient market hypothesis, developed in part by Eugene Fama in the 1960s, argues that all market prices reflect and incorporate all available market information. Thus, knowledgeable traders would disallow bubbles from emerging in the first place (Fama, 1965). In theory, this intuition seems plausible, but it does not have much support from academics or finance professionals.

The Dutch tulip and bulb craze of the early 17th century marks the first recorded market crash. When a non-fatal virus known as mosaic altered tulips in a visually pleasing way, farmers in the Netherlands began to increase tulip supply in an attempt to tap into this speculatively lucrative market. Consequently, the demand and price for tulips increased drastically over a three-month period. As prices climbed, a small percentage of myopic investors began to sell off tulips in an attempt to lock in their capital gains. This trend continued, and inflated prices and irrational consumer behavior led to a drastic tulip selloff and a severe price crash. After a few months, tulips were worthless in price ("Market Crashes: The Tulip and Bulb Craze," 2010).

Fast forwarding to the late 1990s, the Internet bubble wiped away the capital gains of millions of investment portfolios in the United States due to massive overvaluations of emerging technology companies. Following the commercial introduction of the Internet in 1995, many Internet and technology companies surfaced with promising business propositions. A huge wave of ferociously valued IPOs swept market participants into a euphoric state of investing. For example, in 1999, 117 of the 457 IPOs doubled in price on the first day of trading. As time went on, however, many of these companies reported massive losses, and investors realized that these companies lacked solid business plans. From 2000 to 2002, overzealous investors saw their capital gains turn negative ("Market Crashes: The Dotcom Bubble," 2010). This market crash spurred a tremendous amount of literature concerning stock market bubbles.

John Maynard Keynes, the first economic theorist who discussed the possibility of stock market bubble formation, identified irrational investors as the sole cause for equity market bubbles (Keynes, 1936). The majority of 21st century academic literature supports the existence of bubbles. "Bubbles and Crashes," a published article written by two Princeton University professors, suggests that bubbles can form even if rational investors are aware of the deviation between market prices and fundamental prices. The authors argue that it is more rational for an investor to hold assets than it is to sell assets during a surge in prices. Additionally, they argue that it is riskier to ignore the possibility of a bubble than it is to buy into the bubble and then sell equities once the bubble begins to burst (Brunnermeier and Abreu, 2009).

There is a vast amount of qualitative and theoretical evidence for bubble behavior in modern markets. Therefore, the primary goal of this thesis is to provide supporting quantitative and empirical evidence regarding the detection of bubbles. It is highly improbable that any one econometric model will be able to detect bubbles without strong underlying assumptions or perfectly sound theoretical underpinnings. This thesis will not indisputably prove the root causes of bubbles, but it will give potential insight on abnormal market conditions. More specifically, the price-to-earnings test focuses on the relationship between the volatility (risk) and market premium (return). Under certain assumptions, an extremely negative relationship between these two market measures hints at potential bubble-forming conditions.

This thesis observes four published bubble tests constructed over the past few decades to examine the existence of bubbles. Although all of these tests are reputable, each test struggles to indisputably distinguish bubbles as the main driver for sharp equity price movements. Additionally, the tests are dependent on strong underlying assumptions. Nonetheless, the combination of these four tests does an ample job of arguing for the existence of bubbles.

This thesis utilizes the S&P 500 composite price-to-earnings ratio to generate a test for bubbles. The data is manipulated based on a model developed by two Aarhus Business School graduates, Bram Weites and Malte von Maravic (2010). A rolling regression is applied to the model in an attempt to detect conditions that are potentially conducive for bubble formation in equity markets. The true critical bubble value' for the estimated coefficient is unknown, so this thesis is unable to indisputably distinguish between absolute bubble periods and potential bubble periods. However, periods of negative statistically significant estimated coefficients hint that the market is behaving abnormally due to an unidentified force.

2 Literature Review

2.1 Introduction

Since the 1960s, there has been an increase in the amount of literature dedicated to asset price bubbles. Generally, economists agree that the bubble aspect of an asset market is quantified as the difference between market prices and the intrinsic values of assets. This literature review will focus on four econometric bubble tests. Three of the papers, Kenneth West's "*A specification test for speculative bubbles*," Yangru Wu's "*Rational bubbles in the stock market: accounting for the U.S. stock price volatility*," and Kenneth Froot and Maurice Obstefeld's "*Intrinsic bubbles: the case of stock prices,*" focus on fundamental stock price estimations as the explanatory aspect of bubbles. The fourth paper, "*Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values?*" written by Peter Phillips, Yangru Wu, and Jun Yu, focuses on explosive price movements of equities as the driver for bubbles. These four econometric bubble tests will be observed, analyzed, and critiqued in order to advance the goal of this thesis.

2.2 West's (1987) Bubble Test

Kenneth West bases his test for speculative bubbles on the fundamental values of equities, and he calculates these fundamental values as the discounted present value (DPV) of dividends. West's test focuses on 20th century Standard and Poor's Composite Stock Price Index and the Dow Jones Index. West aggregates annual price and dividend data from these two indices to empirically test his theoretical underpinnings.

West presents two null hypotheses regarding stock market price determination. The first null hypothesis test assumes that the efficient markets model determines equity prices. In this model, the future expected streams of dividend payments, subject to a discount rate, determine fundamental stock prices. The second hypothesis assumes that the combination of the efficient markets model and a speculative bubble component determine stock prices. Theoretically, West states that these two DPV valuations should be the same under the conditions of the first null hypothesis. Thus, he is stating that under the efficient markets model, the DPV should be the same under the assumption that fundamentals drive stock prices. Under the second null hypothesis, however, a deviation between the two DPV calculations could hint at an external force driving equity prices (an intrinsic bubble).

West calculates the expected discounted present value of stocks using two different valuation methods. First, he regresses stock prices on the lagged dividend payments. Next, he uses a discount rate (determined using a complex equation) and an ARIMA equation to estimate the future dividend payments. West carried out the testing, and his results hint at the existence of bubbles. The coefficients contingent on the first null hypothesis are statistically different from the coefficients contingent on the second null hypothesis. The equations are very complex, but the incompatibility between the two null hypotheses draws the conclusion that bubbles do exist according to his test.

This paper received a fair amount of criticism. The major area of disconnect in the academic community pertains to the validity of West's DPV calculations. Refet Gürkaynak, a professor at Bilkent University, points out that West runs his tests on both the levels and differences of data. According to Gürkaynak, this skews the interpretation of his coefficients and makes it hard to detect the stationarity of the data. Stationary data is key considering the methods that West uses to forecast future dividend processes. Also, Gürkaynak explains that investors use more information, such as company financials and revenue growth factors, to forecast future dividends. West only uses past dividend payments to forecast future dividends, which is a shortfall when considering realistic market participation (Gürkaynak, 2008).

Other criticisms focus on the econometric process West uses to generate the coefficients. In a Washington, D.C. World Bank research article, Dezhbakhsh & Demirguc-Kunt (1990) criticize west for using the Hausman test. They conclude that this form of testing is too lenient with rejecting the null hypothesis in small samples. Finally, a vast amount of economists and econometricians question the applicability of West's equations to actual market behavior. Some critics view West's dividend forecasting equations as poor indicators for observed dividend stream determination.

The issue of calculating the true fundamental value of stock prices is a recurring theme seen in bubble literature. However, West's model and empirical testing hint that bubbles do exist in equity markets.

2.3 Froot & Obstfeld's (1991) Bubble Test

Froot and Obstfeld base their test for bubbles on fluctuations in stock market prices, and the paper differentiates rational bubbles from intrinsic bubbles. Rational bubbles are caused by exogenous variables, such as macroeconomic factors, while intrinsic bubbles are driven exclusively by the fundamental determinants of asset prices. In other words, exterior economic factors are assumed to be endogenous in this test. Froot and Obstfeld decide to focus on intrinsic bubbles in their test because of the overbearing amount of information that plays into rational bubbles (Froot and Obstfeld, 1991).

Dividends are the key explanatory variable for fundamental value determination in this test, and the model assumes a nonlinear relationship between market prices and fundamental values of stocks. This nonlinear relationship implies that an intrinsic bubble component explains the overreaction in stock market price movement that is independent from the movement of fundamental values. Froot and Obstfeld use 20th century Standard and Poor's stock price and dividend data for their test.

Froot and Obstfeld deploy a geometric martingale (log dividends) to derive the fundamental values. The null hypothesis of this test examines the relationship between price to dividend ratios, estimated fundamental values, and a bubble component. The null hypothesis is as follows: the price to dividend ratio should equal the calculated fundamental value, and the coefficient on the bubble component should be statistically insignificant. The alternative hypothesis is that the bubble component coefficient is statistically significant, which would signify the existence of a bubble (Froot and Obstfeld, 1991).

Empirical testing of the model proves that the bubble component coefficient is statistically different from zero. The coefficient is positive, suggesting that the bubble component causes market prices to exceed fundamental values. Thus, this model empirically proves the existence of bubbles, subject to the underlying assumptions. Froot and Obstfeld add that stock prices overreact to changes in dividends, which explains why historical stock price volatility has not declined as much as dividend volatility.

The academic community criticized the underlying assumptions and empirical testing of this paper. In a Federal Reserve Bank of Chicago article, Ackert and Hunter (1999) commend this paper for using an inclusively complex method to determine fundamental stock values. However, this article argues that the nonlinear relationship between stock prices and dividend payments is explainable by external factors other than bubbles, such as the management implementation of dividend payouts. Furthermore, two Birkbeck University of London professors, Driffill & Sola (1998), argue that the geometric martingale test does an inadequate job of determining fundamental values of stock prices. They ran an alternative model that did not include a bubble component, and the results fit the data just as well.

Overall, this test presents a model that fits the actual data very well. Also, the intrinsic bubble approach allows for the simplification of the test by focusing solely on internal data. The conclusions of the paper hint that bubbles do exist, but the authors failed to prove that bubbles are the only reason for the nonlinear relationship between equity prices and dividend payments.

2.4 Wu's (1997) Bubble Test

Wu's bubble test is technical and complicated, but he defines a bubble as a positive or negative deviation between market prices and fundamental values. Wu's introduction of negative bubbles, which he defines as time periods of extremely undervalued market prices, adds some interesting analysis to this test. The data is comprised of Standard and Poor's stock price data and inflation-adjusted dividends, and Wu generates his model using a matrix analysis along with a Kalman filtering technique.

After testing the model, Wu observes that the bubble component is a statistically significant component of equity prices at certain points in time. More specifically, he concludes that the bubble component is most likely to be statistically significant during bear and bull markets. Also, the results show that negative bubbles were very common prior to the 1960s, but non-existent after the 1960s.

The primary criticism for Wu's test is that he does not completely explain his methodology. Gürkaynak (2005) specifically criticizes the idea of negative bubbles and states that they can never be negative, but Wu's results suggest otherwise. Also, Gürkaynak states that any deviation between fundamental and market equity values could be due to issues with the model, instead of Wu's bubble component (Gürkaynak, 2005).

Although Wu's inclusion of analysis surrounding negative bubbles gives economists a new way to think about bubbles, the complexity of his model makes the results difficult to interpret. Wu's test does not effectively explain the intuitive reasoning behind bubbles since it is completely data driven. However, Wu's quantitative approach helps bridge the gap between theoretical explanations for bubbles and concrete empirical evidence.

2.5 Phillips, Wu & Yu's (2007) Bubble Test

Phillips, Wu, and Yu focus on price movements of stocks as the key indicator for stock market bubble detection. The authors define bubbles as the synchronized occurrence of explosive stock price behavior and non-explosive dividend growth. The model tests the time series for bubbles by detecting for any statistically significant changes in stock market prices to explosive autogressive behavior. The inflation-adjusted NASDAQ Composite Price Index and the NASDAQ dividend series from 1973 to 2005 comprise the data.

Similar to Wu's paper, the methodologies and techniques are complex. The model utilizes a time series of the logarithm of stock prices and dividends. The null hypothesis implies that there are no explosive roots in the model. The authors use an autoregressive approach and the augmented Dickey-Fuller (ADF) test, and certain parameters, such as a variable for standard Brownian motion, are included in the model. A constant discount rate is assumed in the entirety of this model.

Empirical testing of the model fails to reject the null hypothesis at the 10% significance level. The ADF test produced results that signify explosive behavior in stock prices over the period, but the ADF test failed to detect any explosive behavior with the dividends over the period. Thus, the results suggest that a bubble instance occurred over the period because there is explosive stock price behavior without explosive dividend behavior. Next, a separate asymptotic distribution is applied in coordination with the ADF model to pinpoint exact time period when this situation occurred. The test detects abnormal price behavior from mid 1995 to mid 2001, with a peak in early 2000.

To date, this test econometrically detects the Dot-com bubble better than any other paper. However, two University of Bonn professors, Homm and Breitung (2009), criticize this test for using a constant discount rate over the period. They state that the constant discount rate skews the fundamental value calculations. Also, Homm and Breitung argue that the bubble estimator is downwardly biased and has a very large standard deviation (Homm and Breitung, 2009).

This test is the best to date in terms of detecting the concrete start and end dates of bubbles. One shortcoming of the paper is that it focuses on such a short time period. Nonetheless, the test detected the Dot-com bubble with accuracy, which is a major feat for econometric bubble literature.

2.6 Literature Review Observations

Table 1 gives a summary of the four papers observed in the literature review.

Test	Data Used	Basis of Model	Estimation of Fundamental Values
West (1987)	S&P 500 & DJIA	Efficient market hypothesis	DPV of dividend payments
Froot & Obstfeld (1991)	S&P 500	Fundamental values versus market values	DPV of dividend payments
Wu (1997)	S&P 500	Fundamental values versus market values	Complex Kalman Filtering technique to estimate DPV of dividends
Phillips, Wu & Yu (2007)	NASDAQ	Detecting explosive autoregressive behavior in stock price movements	No fundamental value estimation

Table 1: Literature Review Summary

The four tests in the literature review give ample empirical and theoretical evidence for the existence of bubbles in equity markets. Each test relies on a different model and focuses on a different time period, but they all focus on either the NASDAQ or S&P stock indices. West introduced a dividend growth model to calculate the fundamental valuation for stocks in his model. The following two papers elaborate upon this approach by using variations of the dividend growth model dependent on certain assumptions and variable manipulations. Phillips, Wu, and Yu effectively presented the first accurate bubble detection test, and it is plausible that a more accurate bubble detection test can be formulated.

One of the biggest challenges in the finance world is calculating true fundamental values of companies. The fundamental value, or intrinsic value, of a company should include all tangible and intangible aspects of the business. Various fundamental valuation techniques are used in the finance world, but, generally speaking, fundamental value estimation includes the qualitative aspects of a business, such as the business model, governance practices, target market factors, and the total addressable market, and quantitative aspects, such as ratios and financial statement analyses. Every paper is either criticized for the methodology or underlying assumptions used to calculate these values. A model that does not deal with fundamental values may produce the best bubble test to date, given the vast amount of criticism around previous fundamental valuation techniques.

The exclusion of fundamental price valuation and estimation serves as the key advantage for the test utilized in this thesis. Instead, the price-to-earnings ratio is used to detect bubbles. This data is directly observable and does not require estimation. Chart 2 represents the bubble period(s) detected by the tests in the literature review.

Chart 2: Phillip's, Wu, and Yu (2007) bubble detection results versus the NASDAQ

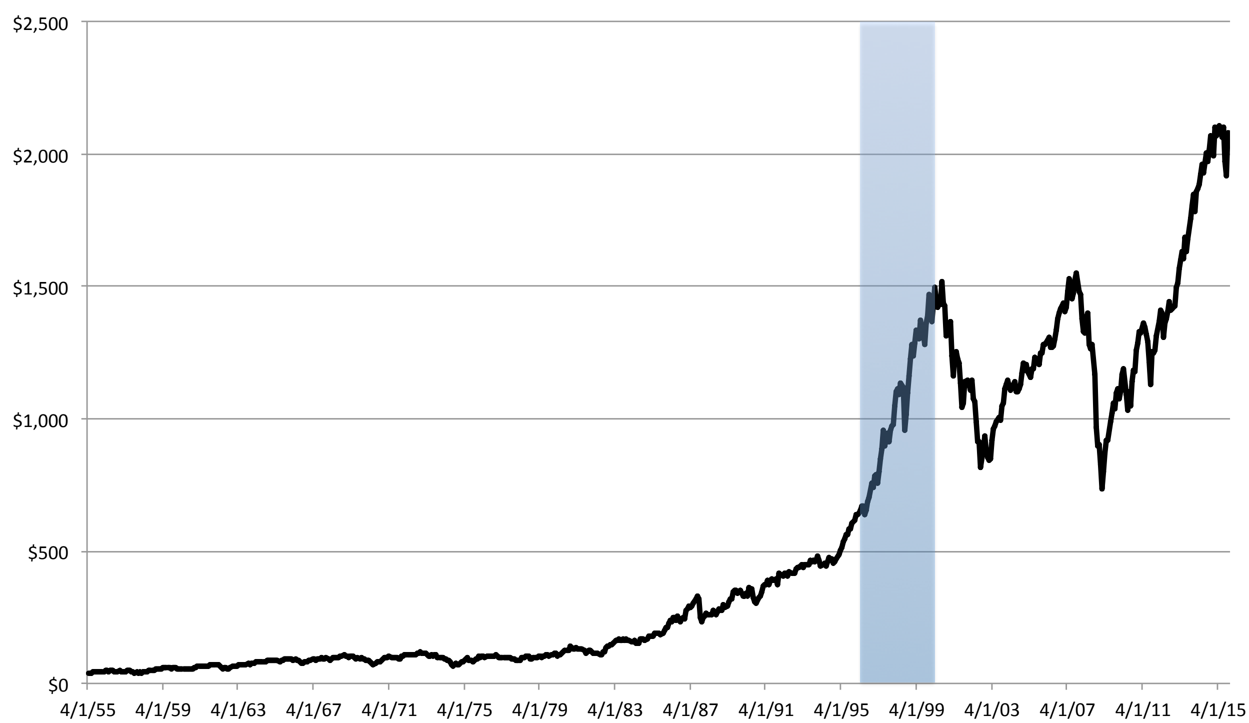

Chart 3: Bubble period detected by Weites and von Maravic's (2010) P/E ratio test

It is difficult to generate precise results for bubble detection tests. Chart 2 shows a bubble detected from June 1995 to July 2001 as predicted by the Phillips, Wu, and Yu (2007) test. Chart 3 represents the 1996 to 2000 bubble as predicted by Weites and von Maravic's (2010) price-to-earnings ratio test. Chart 3 will serve as the preliminary results for the model used in this thesis. The model will be modified and applied to the most current data in an attempt to generate original results.

3 Analytical Foundations

3.1 Introduction

This thesis utilizes a test adapted from a paper written by Bram Weites and Malte von Maravic (2010) that focuses on the price-to-earnings ratio. The test is cemented around an observed relationship between stock market prices and company earnings. Perez explains that stock prices lost their connection to earnings during the late 1990s stock market bubble (Perez, 2009). Given the composition of the ratio, price-to-earnings ratios rose drastically during these two time periods, which serves as an indication that the ratio can be used as an effective bubble detection parameter.

In addition to the price-to-earnings ratio, this model uses the Capital Asset Pricing Model (CAPM). CAPM is a famous academic model that calculates the return of an asset based on the risk free rate of return (R_f), the market risk premium (R_m-R_f), and the relative risk of the asset (β). In this model, risk is the sole driver of an individual stock return, since the market determines R_f and R_m. Generally speaking, risk is measured as the covariance between the asset and the market, divided by the variance of the asset. Thus, the variance (or volatility) of the stock is used as the risk measure ("Capital Asset Pricing Model," 2010).

3.2 The Price-to-Earnings Ratio

The price-to-earnings ratio is measured as the market value per share divided by the company earnings per share. Traders, investors, and other financial service participants often use the price-to-earnings ratio as an investment metric. The ratio gives market participants a general idea of how the earnings of a company relate to the number of shares outstanding and the market valuation of those shares. ("Price-Earnings Ratio," 2010).

There is one key criticism that must be addressed about the price-to-earnings ratio. Miller and Modigliani, two reputable economic theorists, argue that the price-to-earnings ratio counts earnings twice. This is true in the case that a company reinvests its earnings to create new earnings in the future, which would lead to double valuation of the earnings (Miller and Modigliani, 1961). Clearly, this model is not flawless. Weites and Maravic propose the idea of using a Price-to-FCF ratio, but there is not enough historical data to build an effective model (Weites and von Maravic, 2010).

3.3 The Model

This model starts with equation [1], which relates the price and earnings of equities.

[1] $$P_t(1+r) = P_{t+1} + D_t = P_t + E_t$$

In this equation, *E* represents the earnings, *P* represents the stock price, *D* represents the dividends, *t* represents the time period, and *r* represents the return on the stock price. One key definition in this model is that earnings determine the future value of the stock price. To incorporate the price-to-earnings ratio in this model, the equation is manipulated as follows:

[2] $$E_t = P_t(1+r) - P_t = P_t r$$

[3] $$r = \frac{Et}{Pt} = \left(\frac{Pt}{Et}\right)^{-1}$$

Using this logic, the conclusion is that the price-to-earnings ratio is the inverse of the stock return. This is assumed to be true during normal market conditions, but untrue during bubble situations based on Perez's analysis (Perez, 2009).

Weites and Maravic introduce the next part of the model by defining the Capital Market Line (Weites and von Maravic, 2010). The Capital Market Line (CML) is the linear relationship between the rate of return of a portfolio, subject to the risk free rate, and the volatility of the portfolio. The slope of the CML represents the additional expected return of the portfolio associated with a one-percentage point increase in the volatility (measured as the variance) of the portfolio. An individual can alter the expected return of their portfolio by diversifying their portfolio into more risky equities ("Unsystematic Risk," 2010).

Individual stocks also have a relationship with the market as a whole. This is known as market correlation, and a well-diversified portfolio can be composed in a way that eliminates unsystematic risk. This model assumes that there is no unsystematic risk.

[4] $$r_s = r_f + \gamma\rho(\sigma_s)^2 + \varepsilon \qquad \varepsilon \sim N(0, (\sigma_\varepsilon)^2)$$

In this equation, γ represents the slope of the CML, and ρ represents the market correlation. However, this equation does not take into consideration the volatility and return of the market. Thus, equation [4] must be combined with CAPM to include these two variables.

$$(r_s - r_f) = \rho\left(\frac{(\sigma_s)^2}{(\sigma_m)^2}\right)(r_m - r_f) = \gamma\rho(\sigma_s)^2 \quad [5]$$

Equation [5] incorporates the market volatility and risk into the model. The portion of the equation denoted as $\rho\left(\frac{(\sigma_s)^2}{(\sigma_m)^2}\right)$ represents the volatility, or systematic risk, of the portfolio in comparison to the market as a whole (Weites and von Maravic, 2010). In other words, this represents the β of the equation, which is represented by equation [6].

$$\rho\left(\frac{(\sigma_s)^2}{(\sigma_m)^2}\right) = \beta \quad [6]$$

Simplifying equation [5] further generates equation [7].

$$\gamma = \frac{(r_m - r_f)}{(\sigma_m)^2} \quad [7]$$

The final equation [8] is acquired by combining equation [3] with equation [4].

$$\left(\frac{Pt}{Et}\right)^{-1} = r_f + \gamma\rho(\sigma_m)^2 + \varepsilon \qquad \varepsilon \sim N(0, (\sigma_\varepsilon)^2) \quad [8]$$

γ should be positive given the assumption that the inverse price-to-earnings ratio is positively related to the risk of the asset during normal market conditions. Theoretically, investors expect a higher return from an asset as the risk of expected earnings for that asset increases. Looking back at equation 3, the inverse price-to-earnings ratio is essentially a proxy for market return in this model. Thus, according to the assumptions of CAPM, an increase in market volatility should correlate to an increase in market returns. Thus, during bubble periods, the positive relationship between the inverse price-to-earnings ratio and the risk (or volatility) of returns is expected not to hold (Perez, 2009). Thus, bubble periods are expected to yield a negative value for γ.

The market return of the S&P 500 composite index is calculated as the annualized monthly return of the index adjusted to inflation using CPI data from the St. Louis Federal Reserve. The calculation is as follows:

$$R_m = \left[\frac{(Index\ Price)_t + (Dividend)_t}{(Index\ Price)_{t-12}}\right] - 1 \qquad [9]$$

Calculating the volatility of the S&P 500 composite market return serves as a variable of uncertainty in this thesis. A sensitivity analysis is applied to the variance calculations in an attempt to provide additional insight to the results. Three different measures are used: a backward-looking approach, a hybrid approach, and a forward-looking approach. The calculations are made on a monthly basis.

The backward-looking approach serves as the most realistic volatility calculation approach, given that market participants only have access to historical data during real-life investment decisions. This volatility is measured as the variance between the past eleven months of market return, including the current month (time t). The following two approaches, the hybrid approach and forward-looking approach, are deployed in an attempt to provide a sensitivity analysis surrounding the uncertainty of the volatility measure in this model. The financial services industry has deployed a massive risk forecasting effort since the Great Recession, and many firms now use stress testing and scenario analyses in an attempt to prepare for future market risk (Prybylski and Campanile, 2009). Thus, these two tests will provide results contingent on this relatively new strategy used in the finance industry. The hybrid approach is calculated as the variance between the past six months, the current month (time t), and the future five months of market return. Finally, the forward-looking approach is measured as the variance between the current month (time t) and the future eleven months of market return.

4. Data

The data for this model is derived from Robert Shiller's cyclically adjusted price-to-earnings ratio dataset. Robert Shiller uses S&P 500 Composite Stock Price Index to generate his dataset. Shiller adjusts both the S&P 500 composite stock price index and the S&P 500 company earnings to inflation using CPI data from the St. Louis Federal Reserve. Originally, the price-to-earnings data is divided by the preceding ten year moving average of the inflation-adjusted company earnings to cyclically adjust the data, but this thesis does not use this cyclical adjustment manipulation (Shiller, 2015). Cyclical adjustment is unnecessary given the nature of the rolling regression.

The price-to-earnings ratio is calculated as the S&P 500 Composite Price divided by the average of the trailing twelve months (TTM) of earnings per share. The data is measured monthly and ranges from January 1914 to December 2015. This comprises 1221 observations.

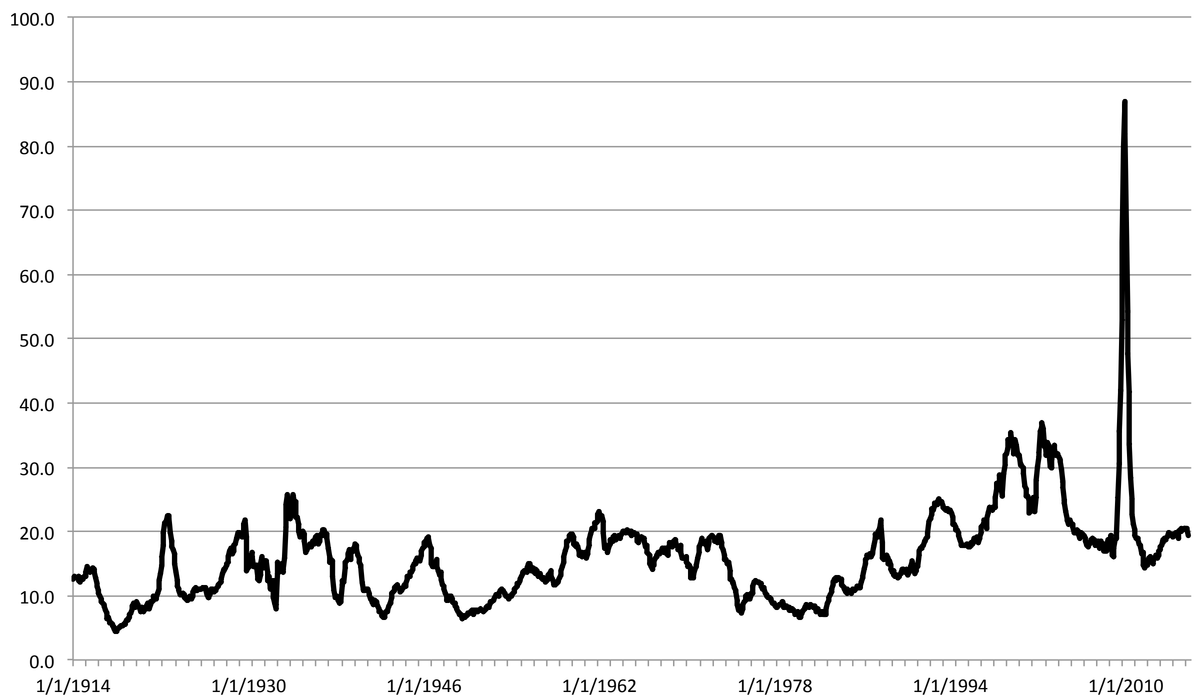

Chart 4: S&P 500 Composite Price-to-Earnings Ratio

The data has a maximum in November 2009 of 86.84 and a minimum in December 1917 of 4.41. The data is relatively flat from 1914 to 1978 and has an upward trend from 1978 to 2014. The spike in the data from 2008 to 2010 is clearly an outlier in this data. There seems to be little to no cyclicality, but there is a noticeable dip in the data during the Great Recession.

The GS 10-year treasury constant maturity rate of the Federal Reserve Bank of St. Louis represents the risk free rate for this model. This 10-year rate is more stable over time, so it fits the model better compared to short-term rates. The data is measured monthly and ranges from January 1881 to December 2015, which comprises 1620 observations.

Chart 5: GS 10-year Treasury Constant Maturity Rate

The data has a maximum in September 1981 of 15.32% and a minimum in July 2012 of 1.53%. The data is relatively flat from 1881 to 1953, but it has an upward trend from 1953 to 1981 and a downward trend from the 1984 to 2015. There is no strong cyclical component to this data.

The S&P 500 Composite Stock Price Index serves as the backbone of this dataset, so the data section includes a graphical representation of the index. The data ranges from January 1881 to December 2015 and is measured monthly. This comprises 1620 observations.

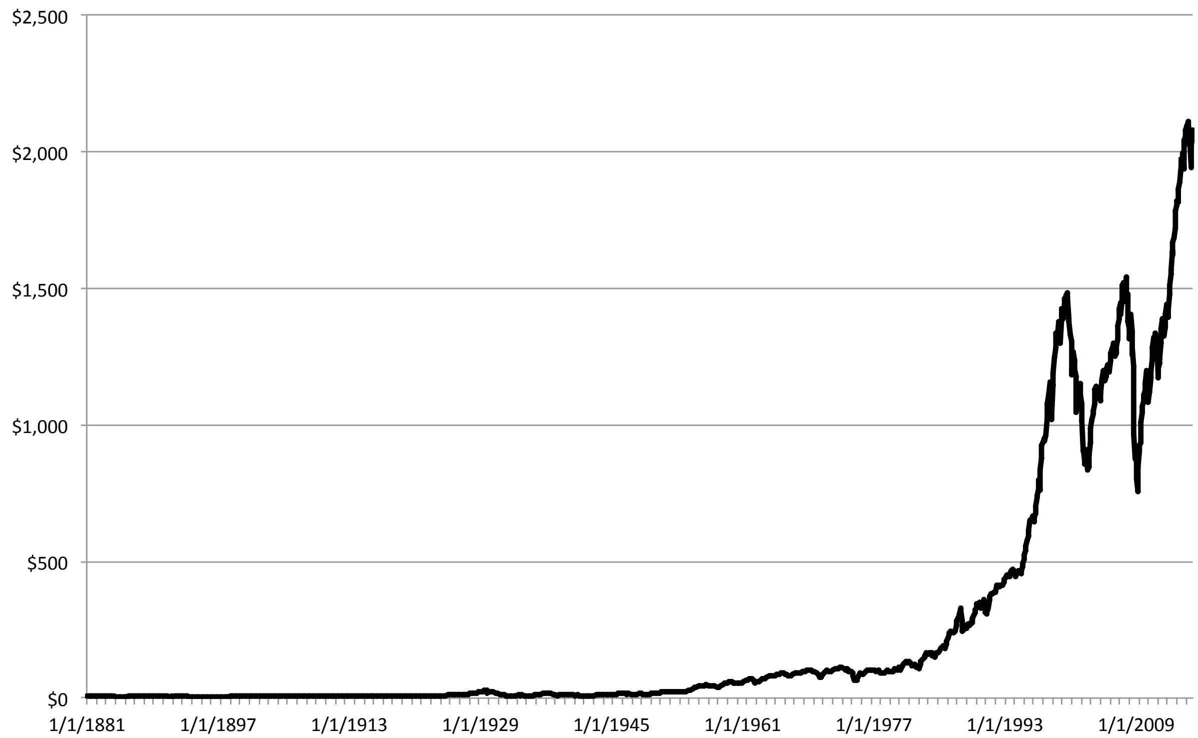

Chart 6: S&P 500 Composite Stock Price Index

The data has a maximum in May 2015 of $2107.39 and a minimum in August 1896 of $3.81. The data is relatively flat from 1881 to 1955, but there are upward trends from 1955 to 2000, from 2002 to 2007, and from 2009 to 2015. There are noticeably sharp downward trends from 2000 to 2002 and from 2007 to 2009, which indicates that there is a strong cyclical component in the second half of the data. Also, the data dips sharply in reaction to the early 2000 recession and during the Great Recession of 2007.

The data is adjusted to inflation using consumer price index data over the same time period to give another visualization of the movement of the S&P 500. The inflation adjustment is applied in order to stay consistent with the literature.

Chart 7: S&P 500 Composite Stock Price Index Adjusted to Inflation

This inflation-adjusted data has a maximum in May 2015 of $2105.55 and a minimum in June 1932 of $83.15. The data has more volatility from 1881 to 1955 compared to the unadjusted S&P 500 data. The trends are very similar to the unadjusted S&P 500 data after 1993, but there are two noticeable peaks that surface in 1929 and 1968 after the inflation adjustment. Also, the inflation-adjusted data seems to be more cyclical.

The first three data trends are compared to detect for any synchronized movement over time. The annualized percent changes are calculated in order to display the three data series on one axis.

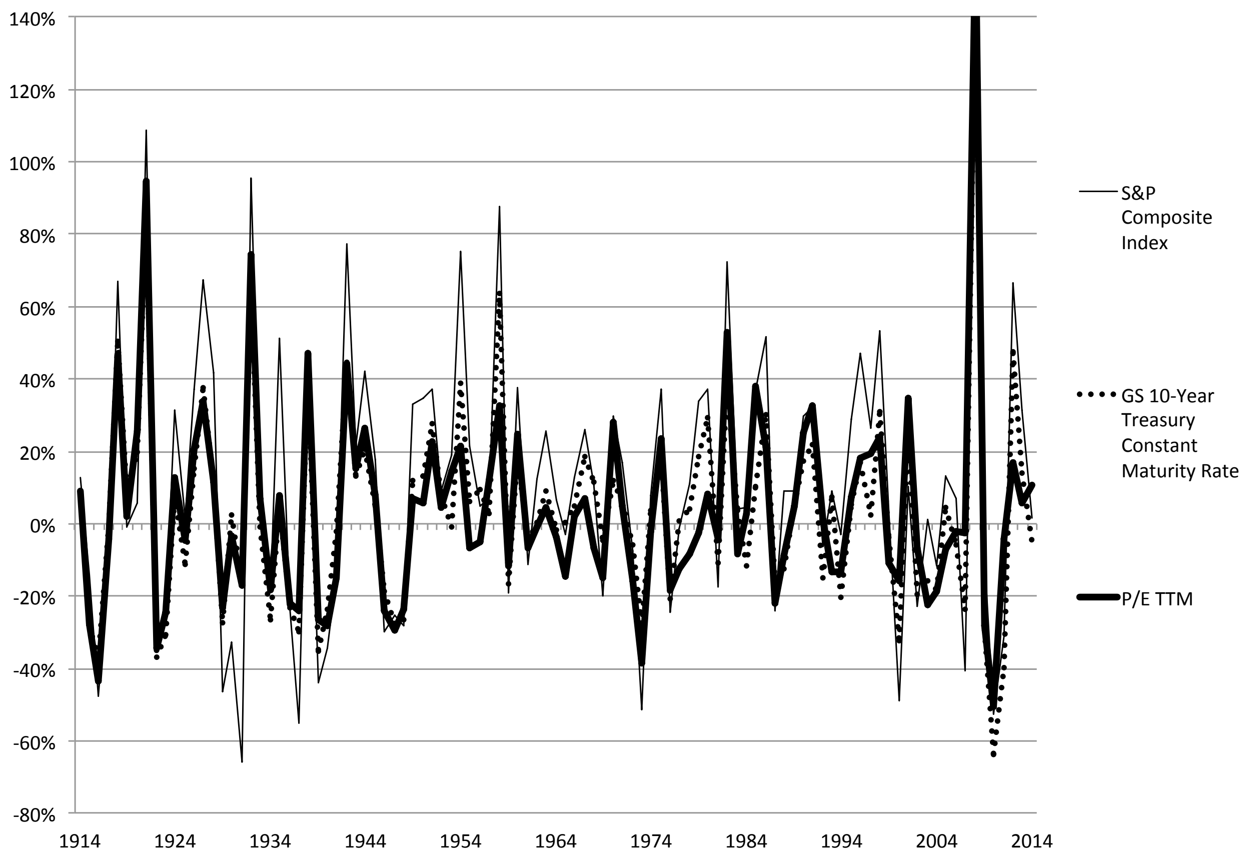

Chart 8: Annualized Percent Change of the Three Data Series

Chart 8 is presented to give a visual representation of the relative volatility between the S&P Composite Index, the GS 10-year Treasury Constant Maturity Rate, and the price-to-earnings ratio TTM. The peaks and troughs are synchronized in many instances, which indicates that the data trends similarly over time. The S&P Composite Stock Price Index has the most volatility over the time period, while the GS 10-Year Treasury Constant Maturity Rate is the most stable. This chart has no explanatory power for bubbles, but it introduces the relative trends of the important data in this model.

5. Methods

Equation [8] serves as the basis for this thesis. The equation is manipulated in order to isolate the return variable from the risk variable of the S&P 500 Composite Index. Regression analysis is applied to equation [10].

[10] $$\left[\left(\frac{Pt}{Et}\right)^{-1} - r_f\right] = \gamma\rho(\sigma_m)^2 + \varepsilon \qquad \varepsilon \sim N(0, (\sigma_\varepsilon)^2)$$

In this regression equation, $(\sigma_m)^2$ is the independent variable and $\left[\left(\frac{Pt}{Et}\right)^{-1} - r_f\right]$ is the dependent variable. The market correlation, or ρ, is assumed to be equal to one because the data used is composite S&P 500 data. Therefore, the correlation between the data and the S&P 500 market as a whole is perfect (or an unadjusted R^2 value equal to 1). The gamma coefficient, γ, measures the relationship between the market premium (using the inverse price-to-earnings ratio as a proxy for market return) and the volatility of the market (using variance as a measure of volatility). As discussed in the Analytical Foundations section of this paper, this relationship is expected to be positive and statistically significant during normal market conditions.

An ordinary least squares (OLS) rolling regression is applied to equation 10 to track the changes in coefficient estimates over time. The time period of the data is set from January 1914 to September 2015, and the rolling regression is conducted using a rolling regression program built for the statistical software called EViews. The program utilizes a moving window rolling regression technique, with a sample size of 60 observations (equivalent to 5 years of data). The sample size is set at 60 observations to ensure that the explanatory power of the estimated coefficients is not trumped by an insufficient amount of data. The step size of the window movement is set to one, which indicates that the regression window shifts forward one month for every additional regression iteration (while maintaining a consistent window size of 60 observations). Finally, the program stores the coefficient estimates (γ) and the corresponding p-values in a new data string. This data is presented and analyzed in the results section of the paper.

6 Results

The rolling regressions are conducted as described in the Methods section, and the results are presented below. Given the nature of the rolling regression, both the estimated coefficients and the corresponding P-values offer valuable insight for analysis.

6.1 Explanatory Power of the Coefficients and P-Values

The rolling regression utilized in this thesis measures the relationship between the risk and the market premium in the S&P 500 Index. Thus, the signs of the coefficients and the corresponding statistical significance levels can be used to detect time periods of theoretical inconsistencies contingent on the assumptions of the model. Negative estimated regression coefficients imply that there is an inverse relationship between market risk and market return. This relationship should be positive during normal market conditions, as discussed in earlier sections of the paper. Time periods of negative estimated coefficients indicate that the market is behaving abnormally, or in other words, there is potential for bubble formation.

In addition to focusing on the estimated coefficients, the P-Values carry significant analytical insight. For the purpose of this thesis, a 95% confidence level will be used as the threshold for statistical significance. The relationship between risk and market premium is statistically significant if the p-value is less than 0.05. However, if the p-value is greater than 0.05, then the relationship is statistically insignificant. In this case, market risk no longer serves as a viable predictive variable for market premium, which also indicates that the market is behaving abnormally.

The estimated coefficients and p-values must be observed together to draw final conclusions. There are four possible combinations of coefficients and p-values, and each combination draws a unique conclusion. The following table summarizes the four potential combinations of estimated coefficients and p-values.

Estimated Coefficient	**P-Value**	**Implication**	**Conclusion**
Positive (+)	Statistically Significant ($P < 0.05$)	Market is behaving normally	No Bubble
Positive (+)	Statistically Insignificant ($P > 0.05$)	Relationship between risk and return is clouded	Potential Bubble Conditions
Negative (-)	Statistically Significant ($P < 0.05$)	Market is behaving abnormally	Bubble
Negative (-)	Statistically Insignificant ($P > 0.05$)	Relationship between risk and return is clouded	Potential Bubble Conditions

Table 9: Estimated Coefficient and P-Value Combinations

6.2 Backward-Looking Volatility Rolling Regression Results

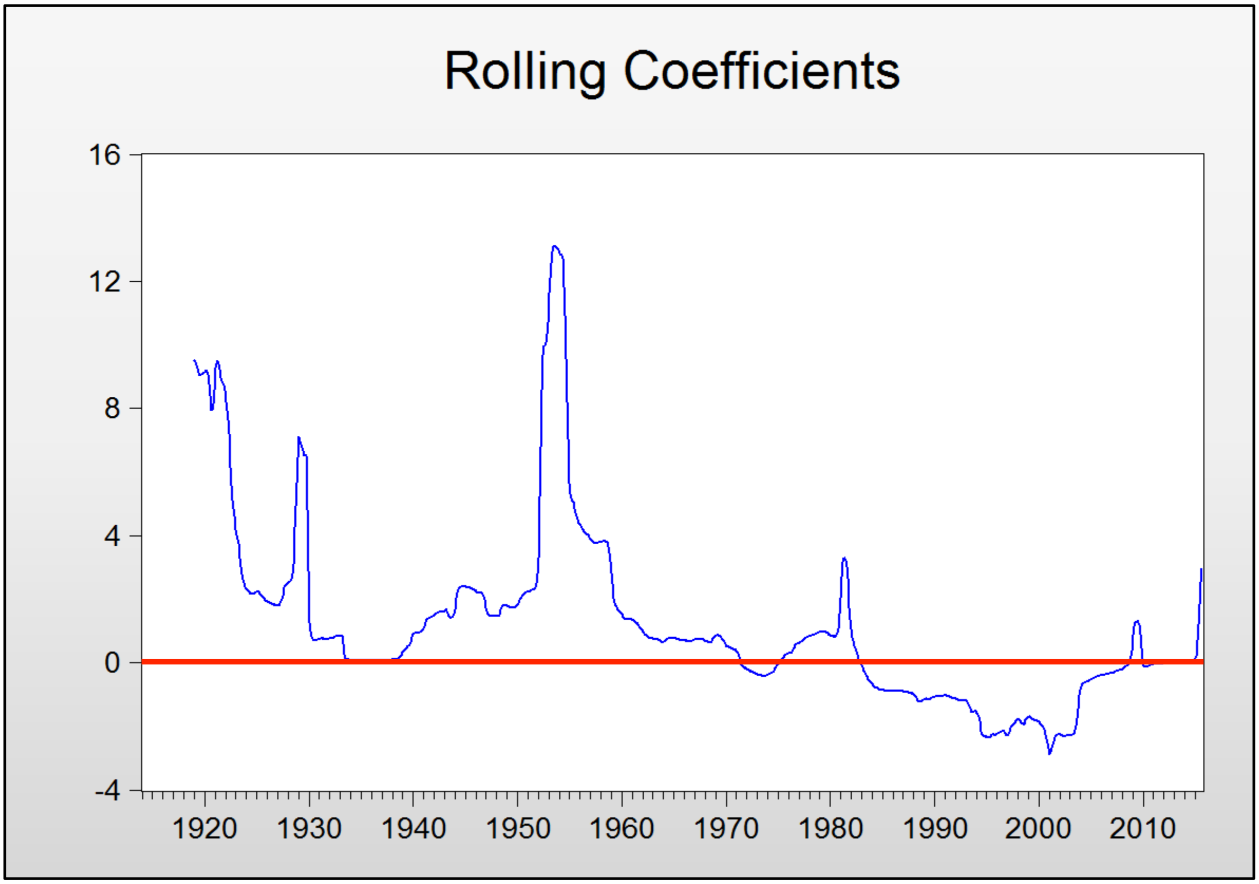

Chart 10: Backward-Looking Volatility Rolling Regression Coefficient Estimates

Chart 10 presents the backward-looking rolling regression coefficient estimates. The coefficient estimates are relatively variable over the time period. Positive estimated coefficients are observed for the entire first half of the data, with noticeable spikes in the 1920s and the 1950s. However, negative estimated coefficients are observed in the early 1970s, and from the 1980s to the early 2000s. These estimated coefficients must be viewed in synchronization with the corresponding p-values in order to draw conclusions.

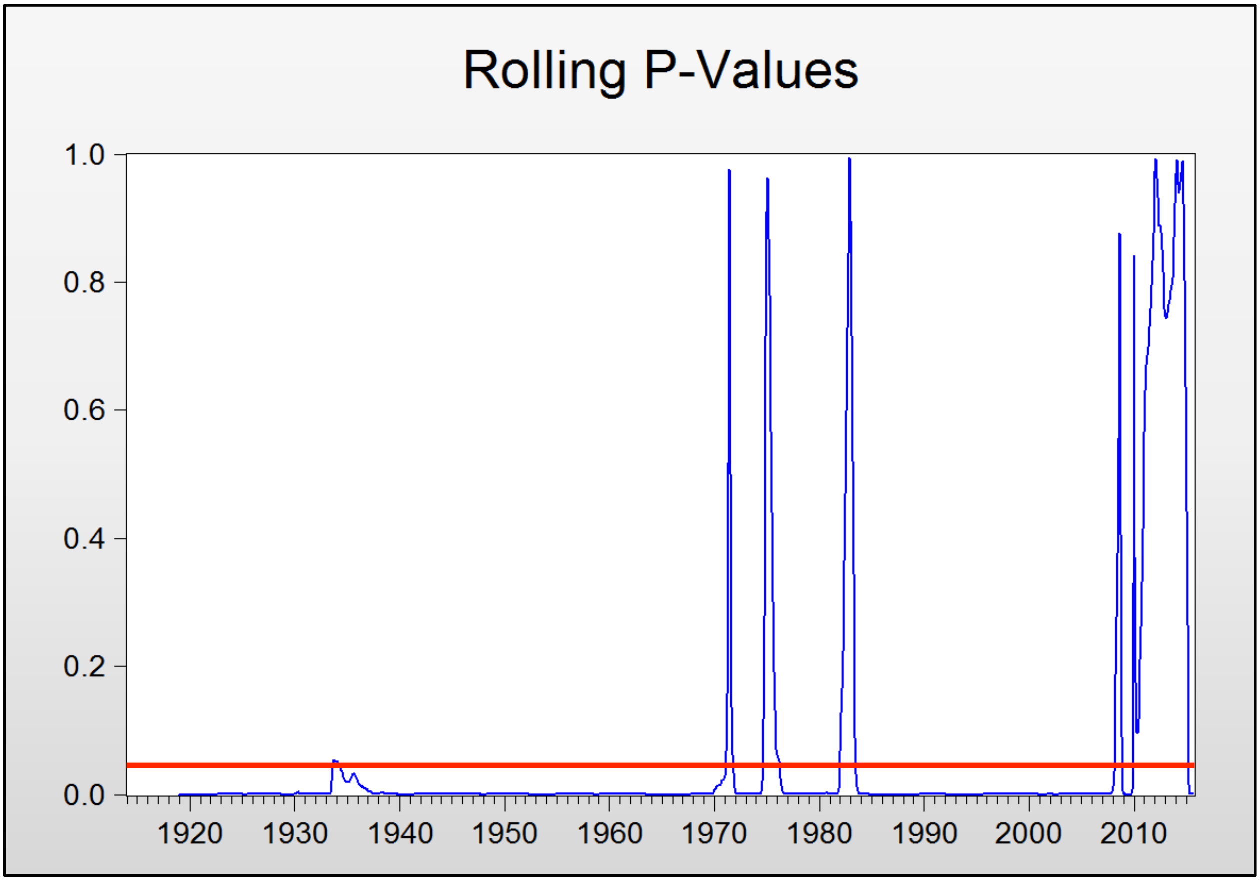

Chart 11: Backward-Looking Volatility Rolling Regression P-Values

The corresponding p-values for the rolling regression coefficient estimates are presented in chart 11. The results are tied to the horizontal axis, but there are many noticeable spikes. For the purpose of this thesis, a 95% confidence level is deployed as the threshold for statistical significance. Thus, p-values that are greater than the critical value of 0.05 are deemed to be statistically insignificant. There are periods of statistical insignificance in the mid-1930s, the early 1970s, the mid-1970s, the early 1980s, and from 2009 to 2015.

6.3 Hybrid Volatility Rolling Regression Results

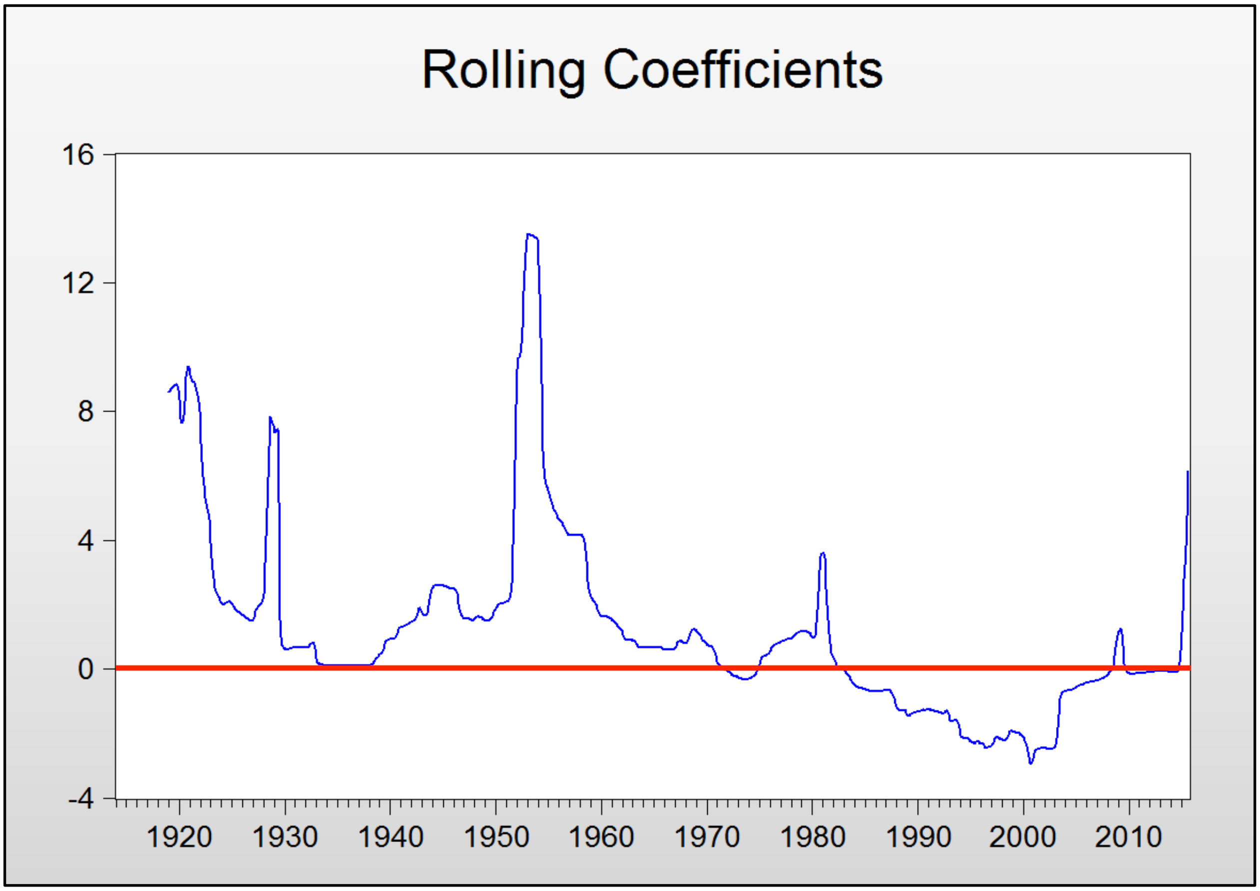

Chart 12: Hybrid Volatility Rolling Regression Coefficient Estimates

Chart 12 presents the hybrid rolling regression coefficient estimates. The coefficient estimates are variable over the time period. Positive estimated coefficients are observed for the entire first half of the data, with noticeable spikes in the 1920s and the 1950s. However, negative estimated coefficients are observed in the early 1970s, and from the 1980s to the early 2000s. These estimated coefficients must be analyzed in tandem with the corresponding p-values in order to draw conclusions.

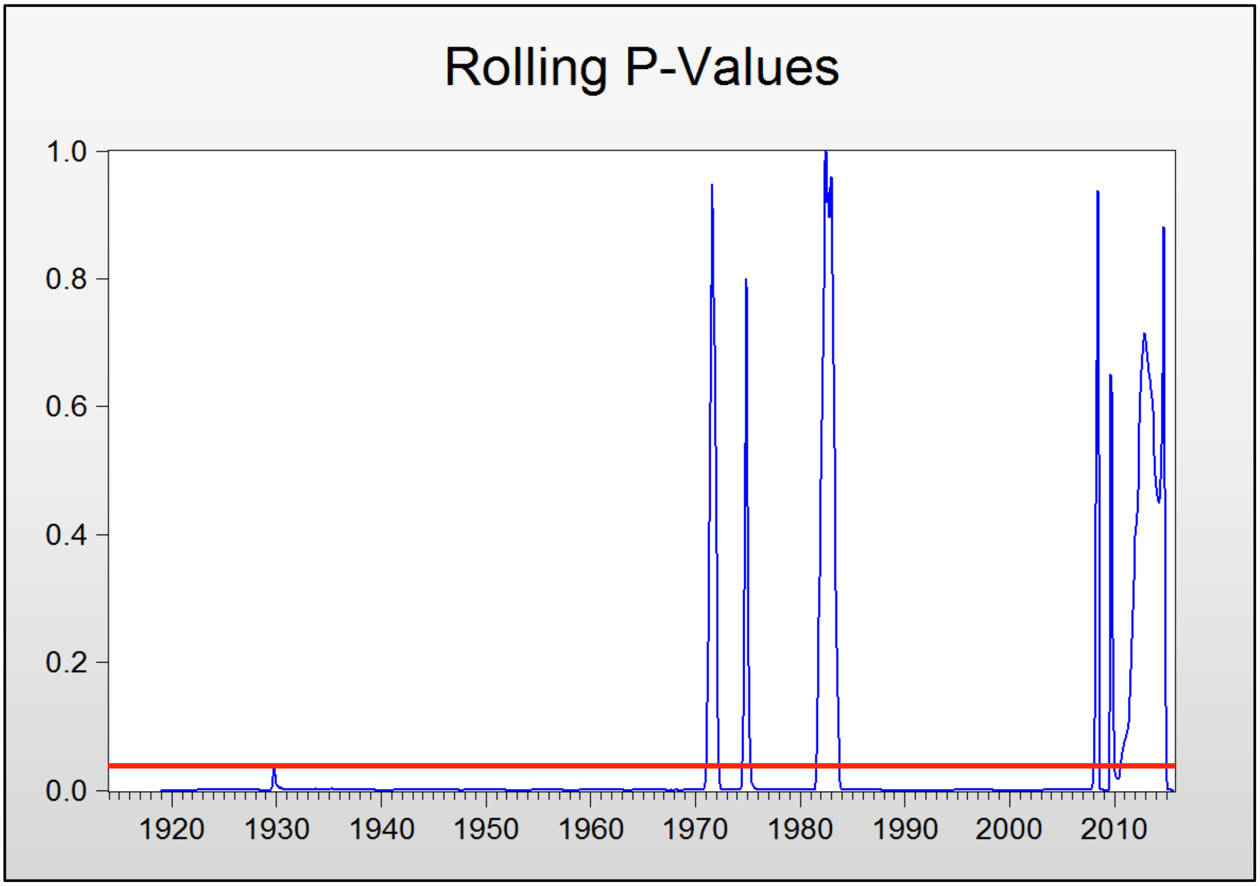

Chart 13: Hybrid Volatility Rolling Regression P-Values

The corresponding p-values for the rolling regression coefficient estimates are presented in chart 13. The results are tied to the horizontal axis, but there are many noticeable spikes. A 95% confidence level is deployed to measure for statistical significance. With the hybrid approach, there are periods of statistical insignificance in the early 1970s, the mid-1970s, the early 1980s, and from 2010 to 2014.

6.4 Forward-Looking Volatility Rolling Regression Results

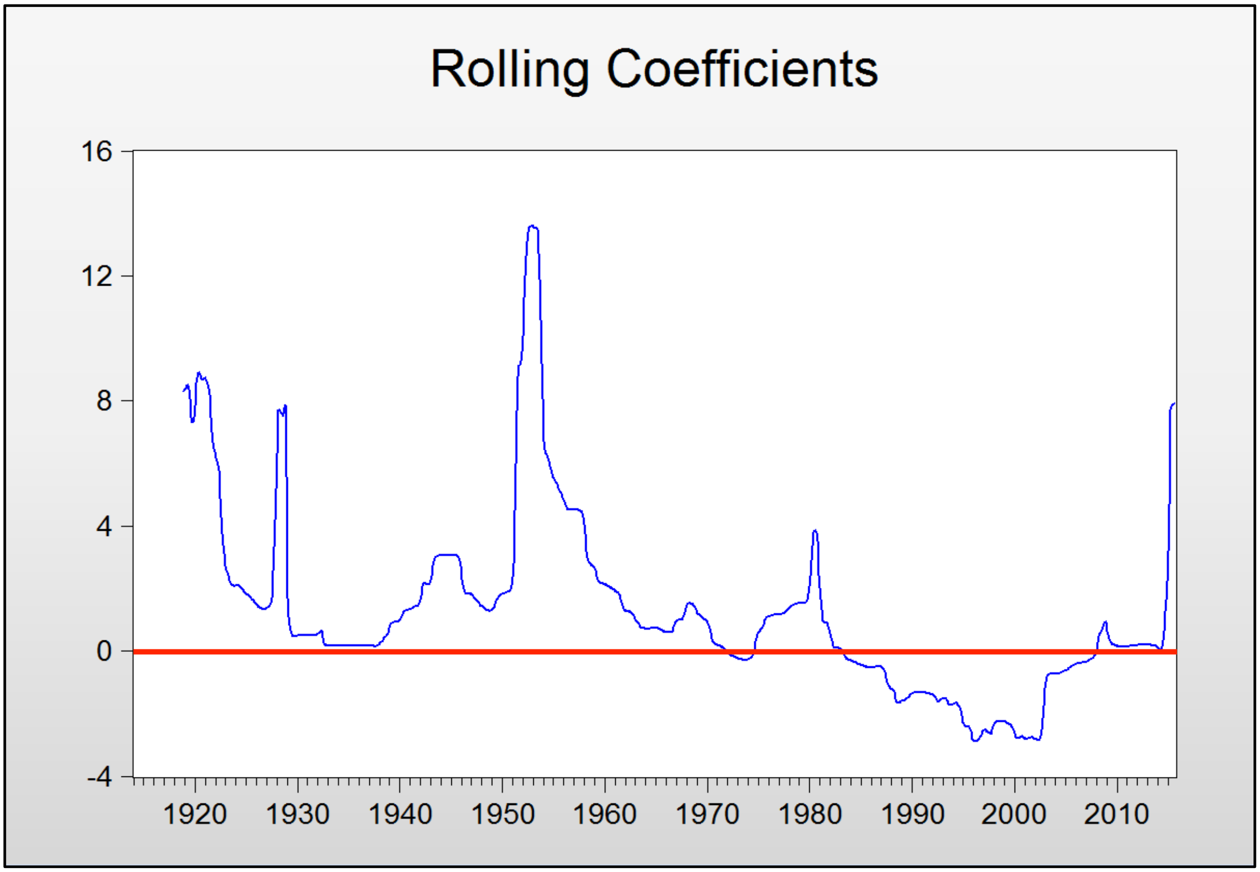

Chart 14: Forward-Looking Volatility Rolling Regression Coefficient Estimates

Chart 14 presents the forward-looking rolling regression coefficient estimates. The coefficient estimates are very variable over the time period. Positive estimated coefficients are observed for the entire first half of the data, with noticeable spikes in the 1920s and the 1950s. However, negative estimated coefficients are observed in the early 1970s, and from the 1980s to the early 2000s. These estimated coefficients must be observed in tandem with the corresponding p-values in order to draw conclusions.

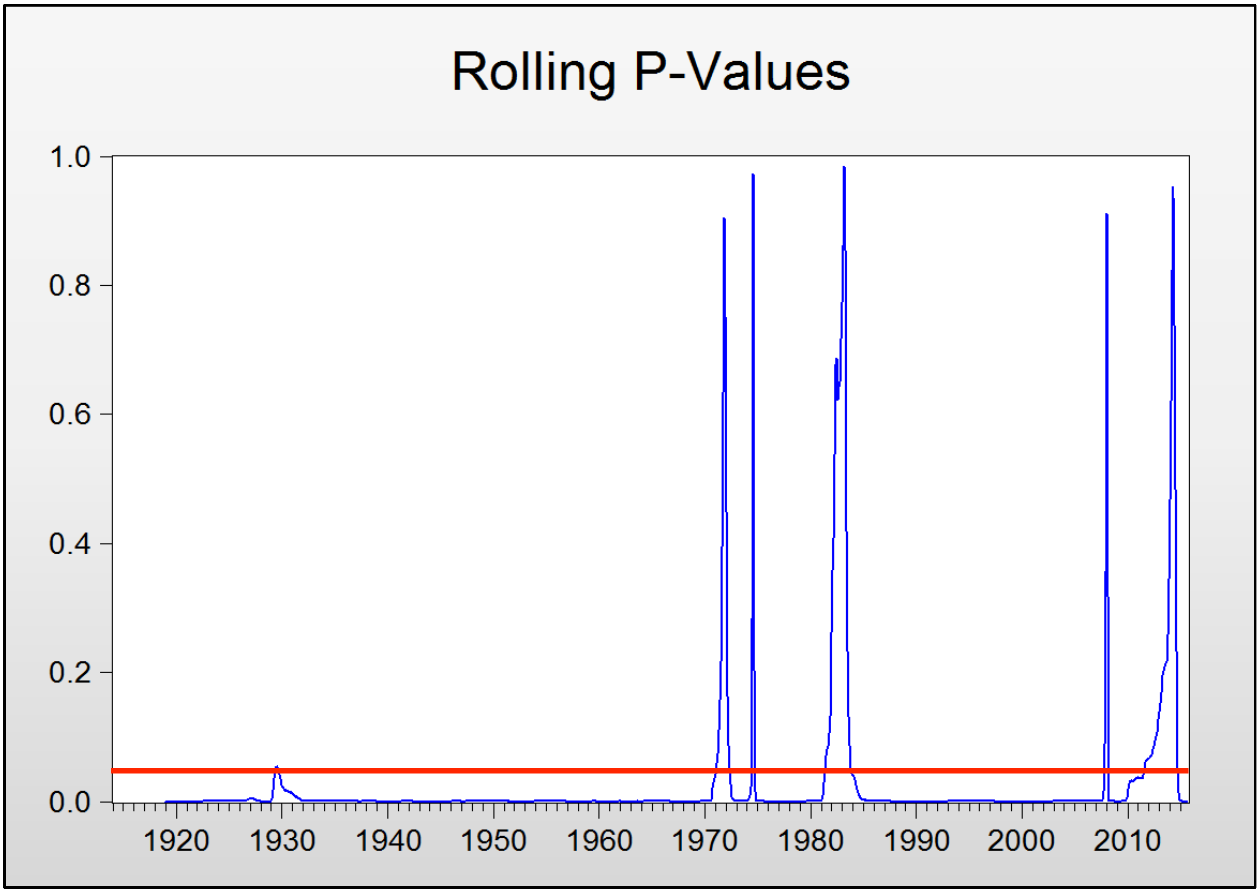

Chart 15: Forward-Looking Volatility Rolling Regression P-Values

The corresponding p-values for the rolling regression coefficient estimates are presented in chart 15. The results are tied to the horizontal axis, but there are many noticeable spikes. A 95% confidence level is deployed to measure for statistical significance. With the future-looking approach, there are periods of statistical insignificance in the late 1920s, the early 1970s, the mid-1970s, the early 1980s, from 2007 to 2008, and from 2011 to 2014.

6.5 Results Summary

The results are summarized in Table 16 below.

Variance Technique	Periods of Statistically Insignificant Coefficients	Periods of Negative Statistically Significant Coefficients	Periods of Very Negative Statistically Significant Coefficients (Coeff < -1.0)
Backward-Looking	September 1933 - February 1934 March 1971- October 1971 September 1974 - March 1976 January 1982 - June 1983 March 2008 - November 2008 December 2009 - March 2015	November 1971 - August 1974 July 1983 - December 1987 January 2004 - February 2008	January 1988 - December 2003
Hybrid	February 1971 - March 1972 August 1974 - March 1975 August 1981 - September 1983 February 2008 - August 2008 August 2009 - December 2009 October 2010 - December 2014	April 1972 - July 1974 October 1983 - November 1987 July 2003 - January 2008 January 2010 - September 2010	December 1987 - June 2003
Forward-Looking	June 1929 - August 1929 February 1971 - May 1972 June 1974 - September 1974 May 1981 - November 1983 November 2007 - February 2008 August 2011 - September 2014	July 1972 - May 1974 December 1983 - August 1987 February 2003 - October 2007	September 1987 - January 2003

Table 16: Summarized Results

7. Conclusions

7.1 Discussion of Results

As seen in Table 16, all three tests present similar and interesting conclusions. There are many time periods of varying length that have statistically insignificant coefficient estimates. These time periods are inconclusive for bubble forming conditions because the coefficient estimates are not statistically different from zero. Thus, we can only conclude that the market is acting abnormally, independent from any bubble activity.

The coefficients for these tests are negative and statistically significant from 1972 to 1974, from 1983 to 1987, and from 2003 to 2008. Thus, given the negative relationship between market risk and market return, we can conclude that some external force is causing abnormal market behavior. This external force is concluded to be a potential bubble based on the assumptions of this thesis. The certainty of a bubble formation is beyond the scope of this thesis, but the final column of Table 16 gives more conclusive insight for bubble formation.

Instead of looking only at the negative coefficient values, Table 16 also specifies when the estimated coefficients are relatively more negative (or less than negative one). Once again, the certainty of bubble formation is impossible to quantify in this test, but looking at statistically significant estimated coefficients that are less than one will allow for relatively more conclusive results. The results indicate that the coefficients are 'very negative' from about 1987 to 2003, with a trough in the early 2000s. These results coincide perfectly with the Dot-com Bubble of the early 2000s, which adds validity to the test.

The relatively wide range of dates must be acknowledged when analyzing these results because it is highly unlikely that the Dot-com bubble spanned nearly sixteen years. This date range is completely contingent on the negative estimated gamma coefficient 'no bubble' threshold set at negative one. A sensitivity analysis is applied to this 'no bubble' threshold in order to hone in on the true Dot-com bubble period. This sensitivity analysis serves as a calibration exercise and is presented in Table 17 below.

'No Bubble' Gamma Threshold Value	Backward-Looking Dot-com Bubble Estimation Period	Hybrid Dot-com Bubble Estimation Period	Forward-Looking Dot-com Bubble Estimation Period
Coefficient Trough	Feb 2001	Oct 2000	May 1996
-1.0	Jan 1988 - Dec 2003	Dec 1987 - June 2003	Sept 1987 – Jan 2003
-1.2	Feb 1993 – Nov 2003	Feb 1998 – May 2003	Jan 1988 – Dec 2002
-1.4	June 1993 – Oct 2003	Jan 1993 – May 2003	Feb 1992 – Dec 2002
-1.6	March 1994 – Sept 2003	Oct 1993 – May 2003	July 1993 – Nov 2002
-1.8	May 1994 – Nov 1997 March 1998 – Oct 1998 Aug 1999 – Aug 2003	Dec 1993 – April 2003	Sept 1994 – Oct 2002
-2.0	July 1994 – May 1997 June 2000 – July 2003	Feb 1994 – Aug 1998 Sept 1999 – March 2003	Nov 1994 – Oct 2002
-2.2	Aug 1994 – March 1996 Sept 2000 – May 2003	Dec 1994 – April 1997 March 2000 – Feb 2003	Dec 1994 – Sept 2002
-2.4	Nov 2000 – July 2001	May 2000 – Jan 2003	Oct 1999 – Sept 2002
-2.6	Dec 2000 – April 2001	July 2000 – Feb 2001	Nov 1995 – Nov 1996 Jan 2000 – Aug 2002
-2.8	Jan 2001 – March 2001	Aug 2000 – Nov 2001	Dec 1995 – June 1996 Jan 2001 – March 2001 Dec 2001 – July 2002
'Best Threshold Value' (γ*)	**-2.4 < γ* < -2.2**	**-2.4 < γ* < -2.2**	**-2.4 < γ* < -2.2**

Table 17: Sensitivity Analysis for the Best 'No Bubble' Threshold Value

The sensitivity analysis presented in Table 17 gives additional insight on the accuracy of the price-to-earnings bubble detection test. The Dot-com bubble detection window becomes tighter as the 'no bubble' gamma threshold values become more negative. This indicates that the value of negative one (-1.0) may not be the best threshold value. Instead, a value closer to negative two (-2.0) may be more appropriate.

As discussed in the Introduction section of this thesis, the Dot-com bubble began after the commercialization of the Internet in 1995 and ended in the early 2000s. More specifically, this bubble started to increase drastically in 1997, climaxed when the NASDAQ hit an all time high in March 2000, and ended when the NASDAQ hit a local trough in late 2002 ("Market Crashes: The Dotcom Bubble," 2010). Thus, the optimal gamma threshold value should detect a bubble that forms roughly between 1995 and 2002.

Table 17 presents the month during which the minimum estimated gamma coefficient occurs for each of the three tests. In all three cases, the minimum gamma coefficient estimate occurs in a month that falls within the observed Dot-com bubble time period. The bottom row of Table 17 indicates the gamma coefficient threshold that optimally detects a bubble during this time period. A gamma threshold coefficient between -2.2 and -2.4 yields the best time period estimation to fit the actual Dot-com bubble for all three tests. Thus, the optimal gamma coefficient threshold, or γ*, is between -2.2 and -2.4 for detecting a bubble with the magnitude of the Dot-com bubble.

An observation must be made about discontinuations in the bubble detection periods. These discontinuations infer that the relative magnitude of the estimated gamma coefficients become slightly less negative during the detected bubble periods. For example, the backward-looking test with a gamma threshold value of -2.2 yields two separate time periods with a gap from 1996 to 2000.

A continuous period that coincides with the optimal gamma threshold value may indicate the test that generates the best results. The forward-looking test with a gamma threshold between -2.2 and -2.4 seems to satisfy both of these conditions. Thus, a concrete threshold value between -2.2 and -2.4 may yield the most accurate results. An additional sensitivity analysis is applied to the forward-looking test in order to determine the most accurate 'no bubble' estimated gamma coefficient threshold value. The results are presented in Table 18 below.

"No Bubble" Gamma Threshold Value	**Forward-Looking Dot-com Bubble Estimation Period**
-2.20	Dec 1994 – Sept 2002
-2.25	Jan 1995 – Sept 2002
-2.30	March 1995 – Sept 2002
-2.35	Jan 1996 – Sept 2002
-2.40	Oct 1999 – Sept 2002

Table 18: Focused Sensitivity Analysis for the Most Accurate 'No Bubble' Threshold Value

Table 17 indicates that the Forward-Looking test with a 'no bubble' gamma threshold value of -2.35 yields a detected bubble that best matches the actual Dot-com bubble data. Thus, the Forward-Looking test with a gamma threshold value of -2.35 is concluded to be the most accurate bubble detection that this thesis is capable of detecting. This detected bubble started in January 1996 and lasted until September 2002.

7.2 Key Takeaways

The model adapted from the paper written by Weites and von Maravic has an advantage over other bubble tests because it does not require the estimation of fundamental stock values. The literature review concludes that this is consistently the most challenging aspect of generating an effective bubble test. Thus, a test focusing on the price-to-earnings ratio should yield more conclusive results.

The results of the price-to-earnings ratio test are conservative. More specifically, this test may fail to detect the smaller bubbles over the time period, which is primarily due to the 'double counting' of earnings. Also, the true non-bubble value of γ is unknown. The sensitivity analysis above implies that values between -2.2 and -2.4 yield the most accurate results. Thus, time periods with estimated gamma coefficients between -2.2 and -2.4 are concluded to be absolute bubble periods. The Forward-Looking test with a gamma threshold value of -2.35 yielded results that matched the actual Dot-com bubble data most accurately.

The early 2000s Internet bubble is detected with high confidence in all three tests. The detection of this bubble is vital since the Dot-com bubble is the largest recorded bubble in U.S. stock market history. The detection of this bubble gives validity to the model and the test used in this thesis.

A few interesting conclusions can be made by looking at the slightly less negative and statistically significant estimated coefficient time periods for the three tests. All three tests detect potential bubble conditions during the first half of the 1970s. There was a stock market crash during this time period, and it is plausible that the relationship between market risk and market return became clouded because of the 1973 to 1975 recession in the United States. OPEC's decision to drastically increase oil prices and the large increase in U.S. government spending for the Vietnam War led to stagflation and recession in the U.S. The synchronized results from these three tests suggest that there likely was a stock market bubble, or at least an impending market correction, during this time period.

A very similar story can be told for the synchronized results indicating potential bubble conditions during the early 1980s. From 1980 to 1982, there were two consecutive recessions in the U.S. that are referred to as 'the double dip recession.' In short, attempts to curb inflation in the U.S. led to tightened monetary policy, which resulted in recession. Once again, the similar results from these three tests suggest that there likely was a stock market bubble, or at least an impending market correction, during this time period.

Finally, all three tests detected potential bubble conditions during the mid-2000s, which coincides with the housing market crash and the Great Recession. The housing bubble had a direct effect on the U.S. economy and the U.S. equity markets. Thus, it is plausible that the residual effects of this asset price bubble caused an equity bubble because of the intricacies between equity markets, mortgage markets, and the housing market.

Together, these three variations of the price-to-earnings bubble test detect four bubbles. The Dot-com bubble is detected with the highest confidence. The next three bubbles, the early 1970s inflation bubble, the double dip bubble, and the housing bubble, are detected with slightly less confidence. The estimated coefficients over these time periods are negative and statistically significant, but the coefficients are not negative enough to detect the bubbles with perfect confidence. However, it is feasible that the relatively larger size of the Dot-com bubble versus the other three bubbles explains the difference in the magnitudes of the negative estimated coefficients. On the other hand, it is plausible that these three bubbles are more recession-driven as opposed to being bubble-driven. Recession-driven market behavior is easily attributed to an impending market correction, which may explain the results. Chart 19 and Chart 20 give a visual representation of the final conclusions of this thesis. The blue shaded areas represent the bubbles detected by this thesis.

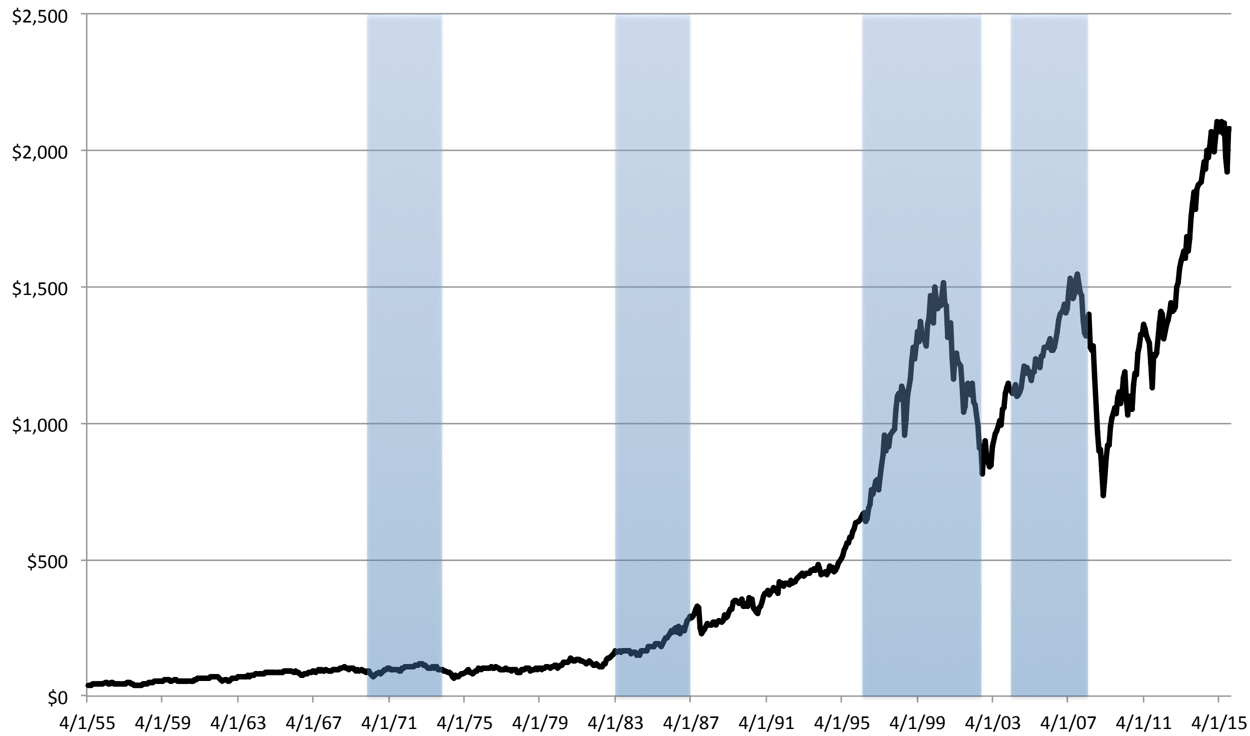

Chart 19: S&P 500 Composite Stock Price Index Adjusted to Inflation

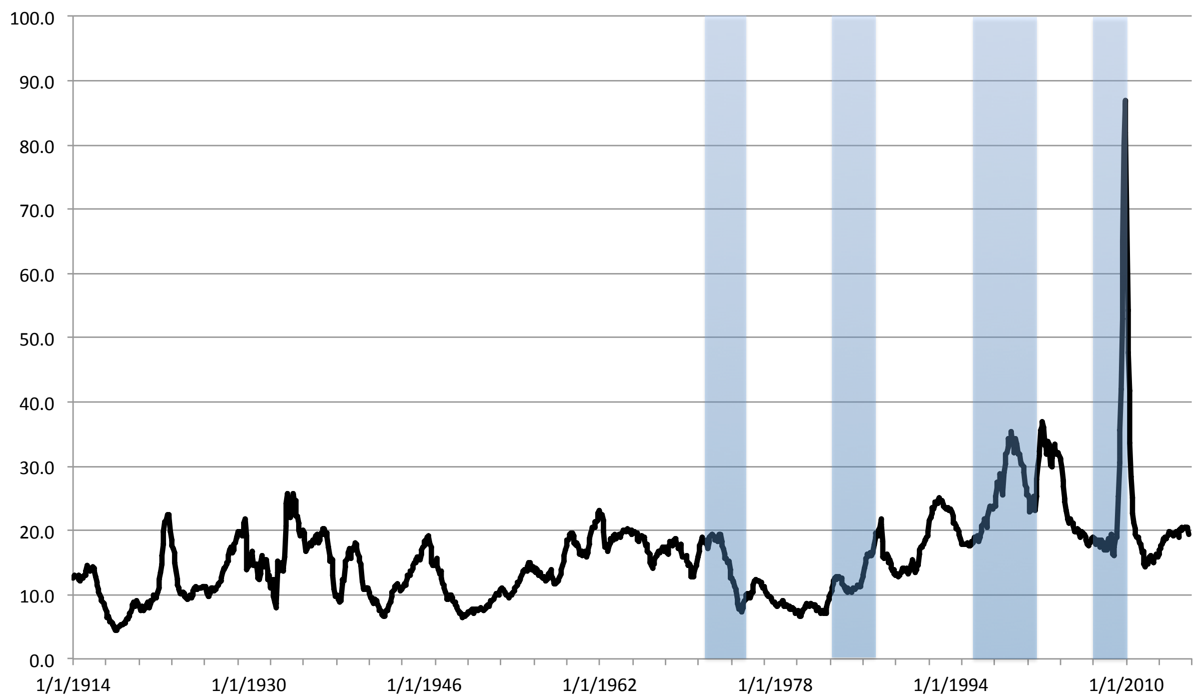

Chart 20: S&P 500 Composite P/E Ratio Adjusted to Inflation

The initial motivation for this thesis was to predict future stock market bubbles, but this task seemed impossible given the unpredictability of the stock market. This thesis does an accurate job of detecting past stock market bubbles by examining the relationship between stock prices, company earnings, market risk, and market return. Although the conclusions of this paper align with previous literature, the approach taken by this paper is unique. A potential extension for this thesis could be formulated using price-to-free cash flow ratio data instead of using price-to-earnings ratio data. Overall, however, the price-to-earnings rolling regression approach proves to be an effective bubble detection tactic.

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